

Implications of non minimal lepton mass textures for Dirac neutrinos

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Abstract

In the light of the recent measurement of the leptonic mixing angle θ_{13} , implications of the latest mixing data have been investigated for non-minimal textures of lepton mass matrices pertaining to Dirac neutrinos. All these texture specific lepton mass matrices have been examined for their compatibility with the latest data in the cases of normal hierarchy, inverted hierarchy and degenerate scenario of neutrino masses. The implications of all the three lepton mixing angles have been investigated on the lightest neutrino mass as well as the Jarlskog's CP violating parameter in the leptonic sector.

1 Introduction

Ever since being proposed by Pauli, neutrinos have been a sort of fascinating puzzle for the physicists. The recent observation of non zero leptonic mixing angle θ_{13} [1]-[4] has provided significant boost to the sharpening of implications of the neutrino oscillations and has added another dimension to neutrino physics by implying the possibility of CP violation in the leptonic sector, hence deepening the flavor puzzle further. The non zero value of θ_{13} , on the one hand, restores the parallelism between the mixings of quarks and leptons, on the other hand, its unexpectedly 'large' value signifies the differences between these, the leptonic mixing angles being large as compared to the quark counterparts.

In the absence of any viable theory of flavor dynamics for explaining the fermion masses and mixings, approaches followed on the theoretical front can broadly be categorized into ‘top-down’ and ‘bottom-up’. Despite large number of attempts using the ‘top-down’ perspective [5], we have yet to arrive at a viable approach which accounts for the vast amount of data related to flavor mixings. Therefore, in the present work, we follow the ‘bottom-up’ approach consisting of finding the phenomenological fermion mass matrices which are in tune with the latest low energy data. In this context, texture specific mass matrices have got a good deal of attention [6] and play an important role in explaining the flavor mixing phenomena. In particular, texture specific mass matrices provide valuable information to explain the quark and lepton masses and mixings, for details we refer the reader to [7]. In the context of leptons, it needs to be mentioned that with the texture approach, minimal textures for Dirac neutrinos have recently [8] been ruled out, however, a detailed and comprehensive analysis for the non-minimal textures in light of the recent measurement of θ_{13} is yet to be carried out.

The purpose of the present work is to carry out a systematic and comprehensive study of non-minimal lepton mass matrices using the texture zero approach. In the context of neutrinos, the issue of these being Dirac like or Majorana particles is still an open question for the physicists since the Dirac neutrinos have not yet been ruled by experimental data. In the present work, considering the neutrinos to be Dirac like particles, we have made an attempt to examine the compatibility of non-minimal lepton mass matrix textures with the recent neutrino mixing data for all the three neutrino mass hierarchies i.e. normal, inverted and degenerate scenario of neutrino masses. It may be noted that while carrying out the analysis, we consider the charged lepton mass matrices and the effective neutrino mass matrices having parallel structures i.e. texture zeroes being located at identical positions, in consonance with some classes of family symmetries and grand unified theories [9] wherein such parallel structures emerge naturally.

The detailed plan of the paper is as follows. In Section (2) we discuss the general lepton mass matrices in the Standard Model (SM). Inputs used in the analysis are

given in Section (3). Results and discussion pertaining to normal hierarchy, inverted hierarchy and degenerate scenario of neutrino masses for texture 2 zero, texture 4 zero and texture 5 zero lepton mass matrices are presented in Section (4). Finally, Section (5) summarizes our conclusions.

2 General lepton mass matrices in the Standard Model

In the Standard Model (SM) [10]-[12] of particle physics, the lepton mass matrices are arbitrary 3×3 complex matrices, thus containing a total of 36 real parameters. Using the ‘Polar Decomposition Theorem’, the lepton mass matrices in SM can be considered to be hermitian without any loss of generality bringing down the number of free parameters from 36 to 18. To facilitate the formulation of phenomenological mass matrices, which perhaps are compatible with the GUT scale mass matrices, it has been suggested [13] that in order to avoid fine tuning amongst the elements of the mass matrices, these should follow a ‘natural hierarchy’ i.e. $(1, 1), (1, 2), (1, 3) \lesssim (2, 2), (2, 3) \lesssim (3, 3)$. The number of free parameters in these mass matrices can further be reduced using the facility of Weak Basis (WB) transformations. These WB approaches broadly lead to two possibilities for the texture zero lepton mass matrices. In the first possibility observed by Branco *et al.* [14], one ends up with a structure wherein one of the matrices is a texture two zero type and the other is a one zero type, i.e.

$$(M_l)_{(1,1)} = (M_l)_{(1,3)} = (M_l)_{(3,1)} = (M_{\nu D})_{(1,1)} = 0, \quad (1)$$

$$\text{or } (M_l)_{(1,1)} = (M_{\nu D})_{(1,1)} = (M_{\nu D})_{(1,3)} = (M_{\nu D})_{(3,1)} = 0, \quad (2)$$

M_l and $M_{\nu D}$ correspond to the charged lepton and the Dirac neutrino mass matrix respectively. In the second possibility, given by Fritzsch and Xing [15], one ends up with a texture two zero structure for the lepton mass matrices, wherein both the

lepton mass matrices assume a texture one zero structure, viz.,

$$(M_l)_{(1,3)} = (M_l)_{(3,1)} = (M_{\nu D})_{(1,3)} = (M_{\nu D})_{(3,1)} = 0. \quad (3)$$

Although the two possibilities are equivalent, however for the present work we have followed the Fritzsch-Xing approach, this choice can be justified as the mass matrices obtained by Fritzsch-Xing approach not only exhibit parallel texture structures but can also be diagonalized exactly making the construction of the corresponding lepton mixing matrix easier. Therefore, the general lepton mass matrices within the framework of SM can be given as

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} C_\nu & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \quad (4)$$

with $A_{l(\nu)} = |A_{l(\nu)}|e^{i\alpha_{l,\nu}}$ and $B_{l(\nu)} = |B_{l(\nu)}|e^{i\beta_{l,\nu}}$.

Following the methodology connecting the lepton mass matrices to the mixing matrix detailed in [7], one can carry out diagonalization of a general mass matrix M_k by expressing it as

$$M_k = Q_k M_k^r P_k, \quad (5)$$

where Q_k , P_k are diagonal phase matrices given as $\text{Diag}(e^{i\alpha_k}, 1, e^{-i\beta_k})$ and $\text{Diag}(e^{-i\alpha_k}, 1, e^{i\beta_k})$ respectively and M_k^r is a real symmetric matrix. M_k^r can be diagonalized by an orthogonal transformation O_k , e.g.,

$$M_k^{diag} = O_k^T M_k^r O_k \quad (6)$$

which can be rewritten as

$$M_k^{diag} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \quad (7)$$

The elements of the general diagonalizing transformation can figure with different

phase possibilities, however, these possibilities are related to each other through the phase matrices [7]. For the present work, we have chosen the possibility

$$O_k = \begin{pmatrix} O_k(11) & O_k(12) & O_k(13) \\ O_k(21) & -O_k(22) & O_k(23) \\ -O_k(31) & O_k(32) & O_k(33) \end{pmatrix}, \quad (8)$$

where

$$\begin{aligned} O_k(11) &= \sqrt{\frac{(E_k - m_1)(D_k + E_k - m_1 - m_2)(D_k + E_k - m_1 - m_3)}{(D_k + 2E_k - m_1 - m_2 - m_3)(m_1 - m_2)(m_1 - m_3)}} \\ O_k(21) &= \sqrt{\frac{(m_1 - C_k)(m_1 - E_k)}{(m_1 - m_2)(m_1 - m_3)}} \\ O_k(31) &= \sqrt{\frac{(E_k - m_2)(E_k - m_3)(m_1 - C_k)}{(m_1 - m_2)(m_1 - m_3)(E_k - C_k)}} \\ O_k(12) &= \sqrt{\frac{(E_k - m_2)(m_3 - C_k)(m_1 - C_k)}{(E_k - C_k)(m_1 - m_2)(m_3 - m_2)}} \\ O_k(22) &= \sqrt{\frac{(E_k - m_2)(C_k - m_2)}{(m_1 - m_2)(m_3 - m_2)}} \\ O_k(32) &= \sqrt{\frac{(-E_k + m_1)(C_k - m_2)(E_k - m_3)}{(m_1 - m_2)(m_2 - m_3)(E_k - C_k)}} \\ O_k(13) &= \sqrt{\frac{(-C_k + m_1)(-E_k + m_3)(C_k - m_2)}{(m_1 - m_3)(m_3 - m_2)(C_k - E_k)}} \\ O_k(23) &= \sqrt{\frac{(-m_3 - C_k)(E_k - m_3)}{(m_1 - m_3)(m_3 - m_2)}} \\ O_k(33) &= \sqrt{\frac{(E_k - m_1)(E_k - m_2)(m_3 - C_k)}{(C_k - E_k)(m_1 - m_3)(m_3 - m_2)}}, \end{aligned} \quad (9)$$

$m_1, -m_2, m_3$ being the eigen values of M_k .

Using the above expressions, one can easily obtain the elements of O_l , the diagonalizing matrix for the charged lepton sector by replacing $m_1, -m_2, m_3$ with $m_e, -m_\mu, m_\tau$, e.g., the first element of O_l can be given as

$$O_l(11) = \sqrt{\frac{(E_l - m_e)(D_l + E_l - m_e - m_\mu)(D_l + E_l - m_e - m_\tau)}{(D_l + 2E_l - m_e - m_\mu - m_\tau)(m_e - m_\mu)(m_e - m_\tau)}}. \quad (10)$$

In an analogous manner, diagonalizing matrix $O_{\nu D}$ for Dirac neutrinos for the normal mass hierarchy (NH) given by $m_{\nu 1} < m_{\nu 2} \ll m_{\nu 3}$ and for the corresponding degenerate scenario given by $m_{\nu 1} \lesssim m_{\nu 2} \sim m_{\nu 3}$ can be obtained from equation (9) by replacing $m_1, -m_2, m_3$ with $m_{\nu 1}, -m_{\nu 2}, m_{\nu 3}$. For instance, first element of $O_{\nu D}$ can be given as

$$O_{\nu D}(11) = \sqrt{\frac{(E_\nu - m_{\nu 1})(D_\nu + E_\nu - m_{\nu 1} - m_{\nu 2})(D_\nu + E_\nu - m_{\nu 1} - m_{\nu 3})}{(D_\nu + 2E_\nu - m_{\nu 1} - m_{\nu 2} - m_{\nu 3})(m_{\nu 1} - m_{\nu 2})(m_{\nu 1} - m_{\nu 3})}}. \quad (11)$$

Similarly, one can obtain the elements of diagonalizing transformation for the inverted hierarchy (IH) case defined as $m_{\nu 3} \ll m_{\nu 1} < m_{\nu 2}$ and the corresponding degenerate case given by $m_{\nu 3} \sim m_{\nu 1} \lesssim m_{\nu 2}$ by replacing $m_1, -m_2, m_3$ with $m_{\nu 1}, -m_{\nu 2}, -m_{\nu 3}$ in eqn. (9), e.g., the first element of $O_{\nu D}$ in this scenario can be given as

$$O_{\nu D}(11) = \sqrt{\frac{(E_\nu - m_{\nu 1})(D_\nu + E_\nu - m_{\nu 1} - m_{\nu 2})(D_\nu + E_\nu - m_{\nu 1} + m_{\nu 3})}{(D_\nu + 2E_\nu - m_{\nu 1} - m_{\nu 2} + m_{\nu 3})(m_{\nu 1} - m_{\nu 2})(m_{\nu 1} + m_{\nu 3})}}. \quad (12)$$

The other elements of the diagonalizing transformations for the charged lepton as well as neutrinos can be obtained in a similar way. The lepton mixing matrix, the ‘Pontecorvo-Maki-Nakagawa-Sakata (PMNS)’ matrix [16] can be obtained from these orthogonal transformations using the relation

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \quad (13)$$

where $Q_l P_{\nu D}$ can be taken as $\text{Diag}(e^{-i\phi_1}, 1, e^{i\phi_2})$. The parameters ϕ_1 and ϕ_2 can be considered as free parameters and are related to the phases of mass matrices as

$$\phi_1 = \alpha_{\nu D} - \alpha_l, \phi_2 = \beta_{\nu D} - \beta_l.$$

3 Inputs used for the analysis

Before getting into the details of the analysis, we would like to mention some of the essentials pertaining to various inputs. In the present analysis, we have made use of the results of a latest global three neutrino oscillation analysis [17], in table (1) we present the 1σ and 3σ ranges of the neutrino oscillation parameters.

Parameter	1σ range	3σ range
$\Delta m_{sol}^2 [10^{-5} eV^2]$	(7.32-7.80)	(6.99-8.18)
$\Delta m_{atm}^2 [10^{-3} eV^2]$	(2.33-2.49)(NH); (2.31-2.49) (IH)	(2.19-2.62)(NH); (2.17-2.61)(IH)
$\sin^2\theta_{13} [10^{-2}]$	(2.16-2.66)(NH); (2.19-2.67)(IH)	(1.69-3.13)(NH); (1.71-3.15) (IH)
$\sin^2\theta_{12} [10^{-1}]$	(2.91-3.25)	(2.59-3.59)
$\sin^2\theta_{23} [10^{-1}]$	(3.65-4.10)(NH);(3.70-4.31)(IH)	(3.31-6.37)(NH);(3.35-6.63)(IH)

Table 1: The 1σ and 3σ ranges of neutrino oscillation parameters presented in [17]

While carrying out the analysis, the lightest neutrino mass, m_1 for the case of NH and m_3 for the case of IH, is considered as a free parameter. For all the three possible mass hierarchies of neutrinos i.e. normal, inverted and degenerate scenario, the explored range of the lightest neutrino mass is taken to be $10^{-8} \text{ eV} - 10^{-1} \text{ eV}$, our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π . Although $D_{l,\nu}$ and $C_{l,\nu}$ are free parameters, however, they have been constrained such that diagonalizing transformations O_l and O_ν always remain real.

4 Results and discussions

Before discussing the results, we would like to briefly discuss the possible non-minimal textures which we plan to analyse and discuss. While considering such possibilities, we ignore WB related mass matrix structures, in particular those related through permutations, as in the case of parallel textures these are equivalent. In the case of texture two zero matrices, therefore we have to consider one possibility

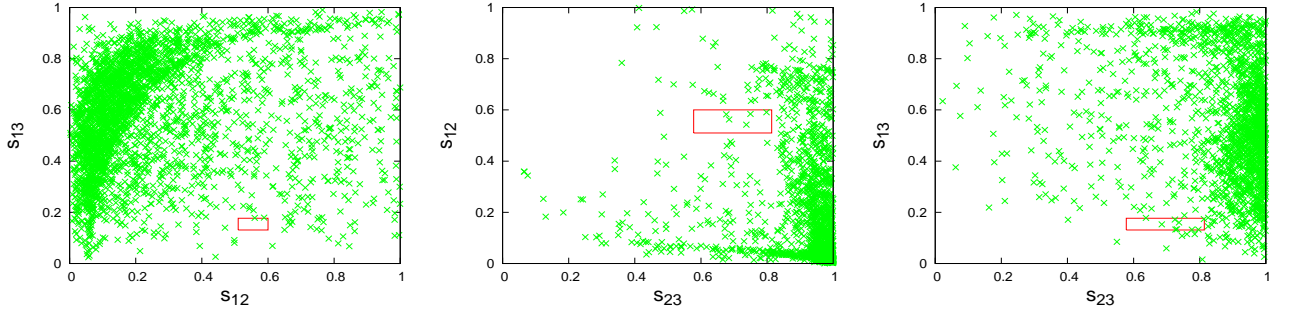


Figure 1: Plots showing the parameter space for any two mixing angles for texture two zero Dirac mass matrices (inverted hierarchy).

only, which we discuss for the case of three hierarchies of neutrino masses. Beyond texture two zero, we have to make specific assumptions of taking particular elements of M_l and $M_{\nu D}$ being zero. In table (2) we have considered four classes with their permutations covering all possibilities. Texture five zero mass matrices can easily be derived from the corresponding texture four zero ones.

4.1 Texture two zero lepton mass matrices

To examine the compatibility of texture two zero lepton mass matrices given in equation (4) with the recent mixing data, we carry out a detailed analysis pertaining to all three possible neutrino mass hierarchies. To this end, in figure (1) we present the plots showing the parameter space of two mixing angles, with the third one being constrained by its 1σ experimental bounds for inverted hierarchy. The rectangular boxes in these figures show the 3σ ranges for the two mixing angles being considered in the figure. As is evident from these figures, the parameter space for the mixing angles shows considerable overlap with the experimentally allowed 3σ region. Therefore, inverted hierarchy seems to be viable for texture two zero lepton mass matrices given in equation (4).

After discussing the viability of inverted hierarchy for texture two zero mass matrices, we now examine the compatibility of these matrices for the normal hierarchy case. To this end, in figure (2) we present the plots showing the parameter space allowed for two mixing angles when the third one is constrained by its 1σ experi-

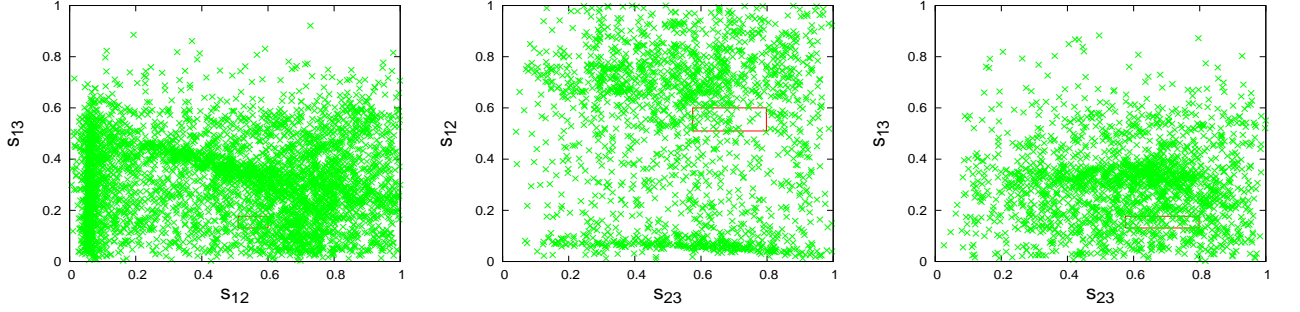


Figure 2: Plots showing the parameter space for any two mixing angles for texture two zero Dirac mass matrices (normal hierarchy).

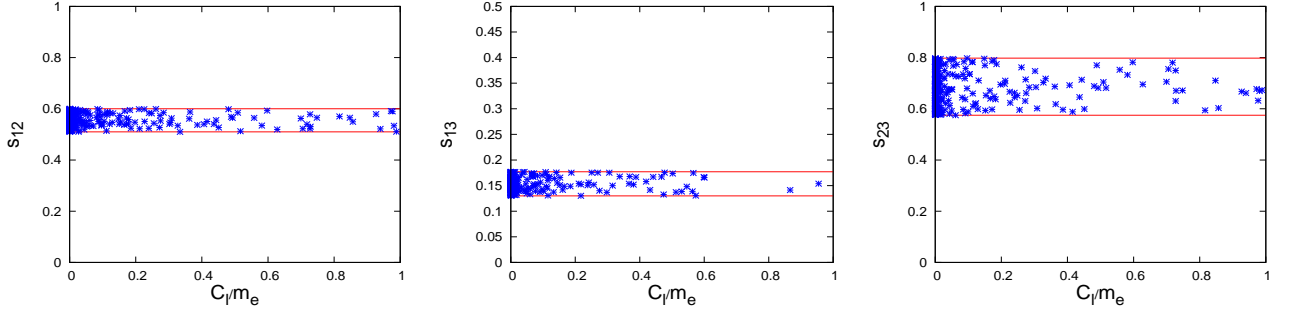


Figure 3: Plots showing the variation of the mixing angles with the (1,1) element of the charged lepton mass matrix for texture two zero Dirac mass matrices (normal hierarchy).

mental bound for normal neutrino mass hierarchy. A general look at the figure (2) reveals that the structure given in eqn.(4) is compatible with the normal neutrino mass hierarchy.

Further, in figures (3) and (4) we present the graphs showing variation of the mixing angles with the parameters C_l/m_e and C_ν/m_1 respectively for structure (4) pertaining to normal hierarchy of neutrino masses. While plotting these figures, all the three mixing angles have been constrained by their 3σ experimental bounds, while all the free parameters have been given full variation. Taking a careful look at these plots it is interesting to note that the leptonic mixing angles donot have much dependence on the parameters C_l and C_ν . Further one can see that a fit for all the three mixing angles can be obtained for the values of parameters being $C_l \lesssim 0.9m_e$ and $C_\nu \lesssim 0.9m_1$.

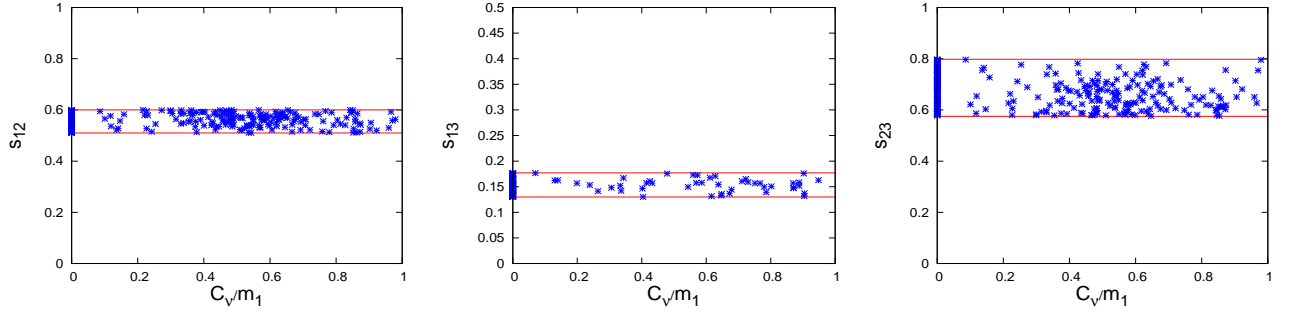


Figure 4: Plots showing the variation of the mixing angles with the (1,1) element of the neutrino mass matrix for texture two zero Dirac mass matrices (normal hierarchy).

To further emphasise this conclusion, we present in figure (5) the plots showing the dependence of leptonic mixing angle s_{13} on C_l/m_e and C_ν/m_3 , constraining the other two angles by their 3σ experimentally allowed ranges for inverted hierarchy of neutrino masses. The two parallel lines in these figures show the 3σ allowed range for the mixing angle s_{13} . Figure (5) clearly shows that in order to accomodate the latest 3σ ranges for the mixing parameters, one requires C_l/m_e and $C_\nu/m_3 \lesssim 1$. Therefore, one can conclude that very small values of the parameters C_l and C_ν are required to fit the latest experimental data for the structure (4) corresponding to both normal as well as inverted neutrino mass orderings and the structure (4) thus essentially reduces to

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix} \quad (14)$$

The structure (14) is referred to as Fritzsch-like texture four zero structure and is studied extensively in literature [18]. However, no such attempt has been made after the recent measurement of the mixing angle s_{13} . Therefore, it becomes interesting to analyse this structure in detail for its compatibility with the latest lepton mixing data.

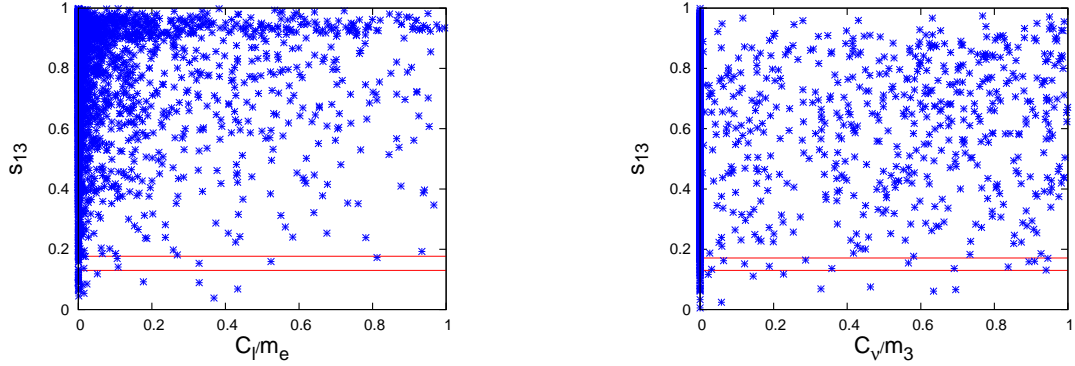


Figure 5: Plots showing the variation of the mixing angle s_{13} with the (1,1) element of the charged lepton and neutrino mass matrices for texture two zero Dirac mass matrices (inverted hierarchy).

4.2 Texture four zero lepton mass matrices

As discussed in the previous section, starting with the most general lepton mass matrices having ‘natural hierarchy’ [13] among their elements, one essentially arrives at the Fritzsch-like texture four zero structure as given in eqn.(14). In this context, it is interesting to note that different four-zero texture parallel matrices, with zeros located in different positions, may have exactly the same physical content. Indeed, they can be related by a WB transformation, performed by a permutation matrix P ,

$$\begin{aligned} m'_l &= P^T m_l P \\ m'_\nu &= P^T m_\nu P \end{aligned} \tag{15}$$

which automatically preserves the parallel structure, but changes the position of the zeros. The matrix P belongs to the group of six permutation matrices, which are isomorphic to S_3 . The four-zero texture ansatz can then be classified into four classes, as indicated in table (2). It can easily be seen that the matrices given in eqn.(14) belong to class I of table (2). In this context, it becomes interesting to confront all the classes of lepton mass matrices with the latest experimental data. From the table, it is clear that class IV is not viable as all the matrices in this class correspond to the scenario where one of the generations gets decoupled from the

	Class I	Class II	Class III	Class IV
a	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & D & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & E \end{pmatrix}$	$\begin{pmatrix} D & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & E \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Be^{i\gamma} \\ Ae^{-i\alpha} & \mathbf{0} & De^{i\beta} \\ Be^{-i\gamma} & De^{-i\beta} & E \end{pmatrix}$	$\begin{pmatrix} A & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & D & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & E \end{pmatrix}$
b	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & E & Be^{i\beta} \\ Ae^{-i\alpha} & Be^{-i\beta} & D \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & E & Be^{i\beta} \\ Ae^{-i\alpha} & Be^{-i\beta} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & De^{i\gamma} & Ae^{i\alpha} \\ De^{-i\gamma} & E & Be^{i\beta} \\ Ae^{-i\alpha} & Be^{-i\beta} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} E & \mathbf{0} & Be^{i\alpha} \\ \mathbf{0} & A & \mathbf{0} \\ Be^{-i\alpha} & \mathbf{0} & D \end{pmatrix}$
c	$\begin{pmatrix} D & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & E \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & D & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & E \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & \mathbf{0} & De^{i\gamma} \\ Be^{-i\beta} & De^{-i\gamma} & E \end{pmatrix}$	$\begin{pmatrix} E & Be^{i\alpha} & \mathbf{0} \\ Be^{-i\alpha} & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A \end{pmatrix}$
d	$\begin{pmatrix} E & Be^{i\beta} & \mathbf{0} \\ Be^{-i\beta} & D & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} E & Be^{i\alpha} & \mathbf{0} \\ Be^{-i\alpha} & \mathbf{0} & Ae^{i\beta} \\ \mathbf{0} & Ae^{-i\beta} & D \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Be^{i\alpha} & Ee^{i\gamma} \\ Be^{-i\alpha} & \mathbf{0} & Ae^{i\beta} \\ Ee^{-i\gamma} & Ae^{-i\beta} & D \end{pmatrix}$	$\begin{pmatrix} A & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & E & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & D \end{pmatrix}$
e	$\begin{pmatrix} D & Be^{i\beta} & Ae^{i\alpha} \\ Be^{-i\beta} & E & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} E & \mathbf{0} & Be^{i\alpha} \\ \mathbf{0} & D & Ae^{i\beta} \\ Be^{-i\alpha} & Ae^{-i\beta} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ee^{i\gamma} & Be^{i\alpha} \\ Ee^{-i\gamma} & D & Ae^{i\beta} \\ Be^{-i\alpha} & Ae^{-i\beta} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & Be^{i\alpha} \\ \mathbf{0} & A & \mathbf{0} \\ Be^{-i\alpha} & \mathbf{0} & C \end{pmatrix}$
f	$\begin{pmatrix} E & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ Be^{-i\beta} & Ae^{-i\alpha} & D \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Be^{i\alpha} & Ae^{i\beta} \\ Be^{-i\alpha} & E & \mathbf{0} \\ Ae^{-i\beta} & \mathbf{0} & D \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Be^{i\alpha} & Ae^{i\beta} \\ Be^{-i\alpha} & \mathbf{0} & Ee^{i\gamma} \\ Ae^{-i\beta} & Ee^{-i\gamma} & D \end{pmatrix}$	$\begin{pmatrix} E & Be^{i\alpha} & \mathbf{0} \\ Be^{-i\alpha} & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & A \end{pmatrix}$

Table 2: Table showing various phenomenologically allowed texture 2 zero possibilities, categorized into four distinct classes.

other two. Such classification of all possible parallel texture four zero structures was first presented in the analysis by Branco *et al.* [14] wherein they carry out detailed study of lepton mass matrices corresponding to all the four classes pertaining to Majorana neutrinos. However, such analysis has not been carried out considering the neutrinos to be Dirac like particles. Therefore, in this section, we carry out a detailed study of all classes of texture four zero lepton mass matrices assuming the neutrinos to be Dirac type.

4.2.1 Class I ansatz

To begin with, we carry out a detailed analysis for texture four zero mass matrices belonging to class I of table (2), i.e.,

$$M_i = \begin{pmatrix} 0 & A_i e^{i\alpha_i} & 0 \\ A_i e^{-i\alpha_i} & D_i & B_i e^{i\beta_i} \\ 0 & B_i e^{-i\beta_i} & E_i \end{pmatrix}, \quad (16)$$

where $i = l, \nu_D$ corresponds to the charged lepton and Dirac neutrino mass matrices respectively. The diagonalizing transformations for this class can be obtained by substituting $C_l = C_\nu = 0$ in eqn. (9). For the purpose of calculations, the elements D_l, D_ν as well as the phases ϕ_1 and ϕ_2 have been considered as free parameters. Following the methodology as discussed in section (2), we attempt to carry out a detailed study pertaining to normal, inverted as well as degenerate neutrino mass orderings. Firstly, we examine the compatibility of mass matrices given in eqn. (14) with the inverted hierarchy of neutrino masses. For this purpose in figures (6) and (7), we present the parameter space corresponding to any two mixing angles while the third one being constrained by its 1σ and 3σ range respectively. The blank rectangular regions in these figures represent the 3σ allowed ranges of the two mixing angles being considered. As can be seen from all these plots, the parameter space of two angles does not show any overlap with the experimentally allowed region. Therefore, we find that the inverted hierarchy seems to be ruled out at both the 1σ as well as the 3σ level for class I ansatz of texture four zero Dirac neutrino mass

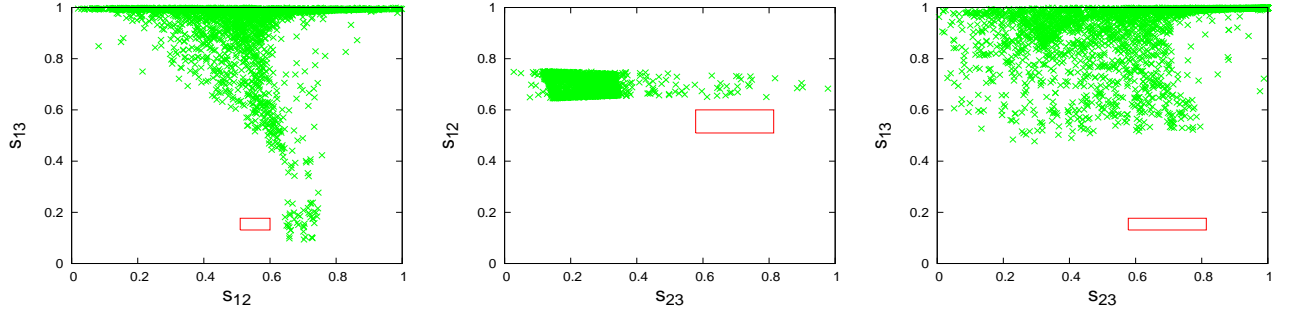


Figure 6: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class I ansatz of texture four zero Dirac mass matrices (inverted hierarchy) .

matrices.

After ruling out the structure given in eqn.(16) for the inverted hierarchy, we now proceed to examine the compatibility of these matrices for the normal hierarchy case. To this end, in figure (8) we present the plots showing the parameter space corresponding to any two mixing angles wherein the third one is constrained by its 1σ range. Interestingly, normal hierarchy seems to be viable in this case as can be seen from these plots, wherein the parameter space shows significant overlap with the experimentally allowed 3σ region shown by the rectangular boxes in each plot. Further, in figure (9) we present the graphs showing the variation of the lightest neutrino mass with the mixing angles for normal hierarchy, keeping the other two mixing angles constrained by their 3σ bounds. The parallel lines in each plot show the 3σ range of the mixing angle being considered. Taking a careful look at these graphs, one can find the range the lightest neutrino mass to be $0.001eV \lesssim m_{\nu 1} \lesssim 0.01eV$ approximately.

Next, in order to explore the possibility of CP violation in the leptonic sector, in figure (10) we study the dependence of Jarlskog's rephasing invariant parameter J on each of the three mixing angles, while keeping the other two mixing angles constrained by their 3σ ranges. From these graphs, there appears to be significant possibility of CP violation in the leptonic sector and the range for the magnitude of J seems to be $0.0005 - 0.03$ approximately.

The degenerate scenario of the neutrino masses can be characterized by either

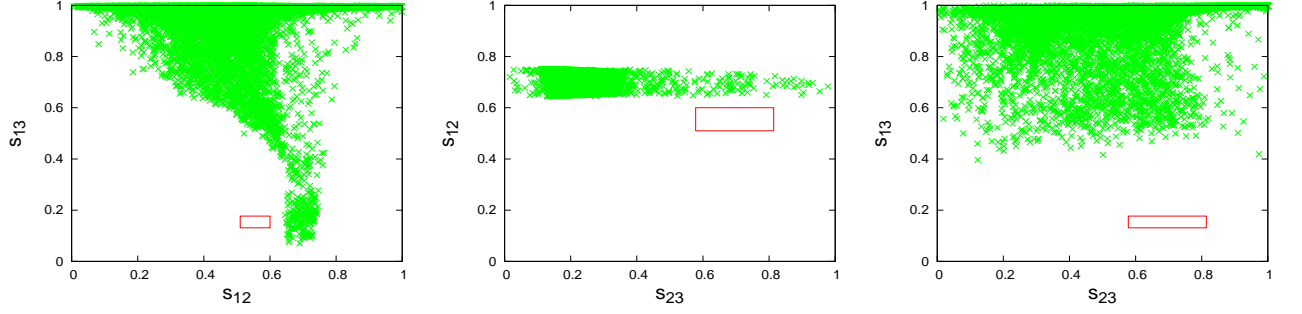


Figure 7: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range for Class I ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

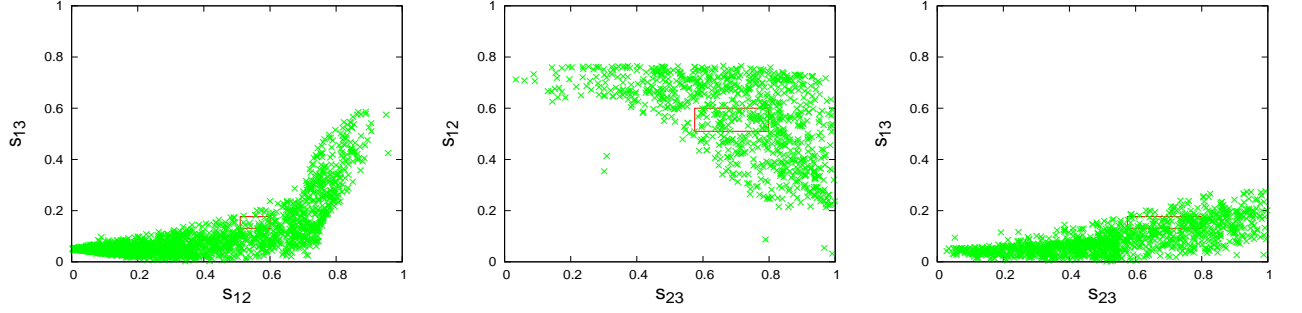


Figure 8: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class I ansatz of texture four zero Dirac mass matrices (normal hierarchy) .

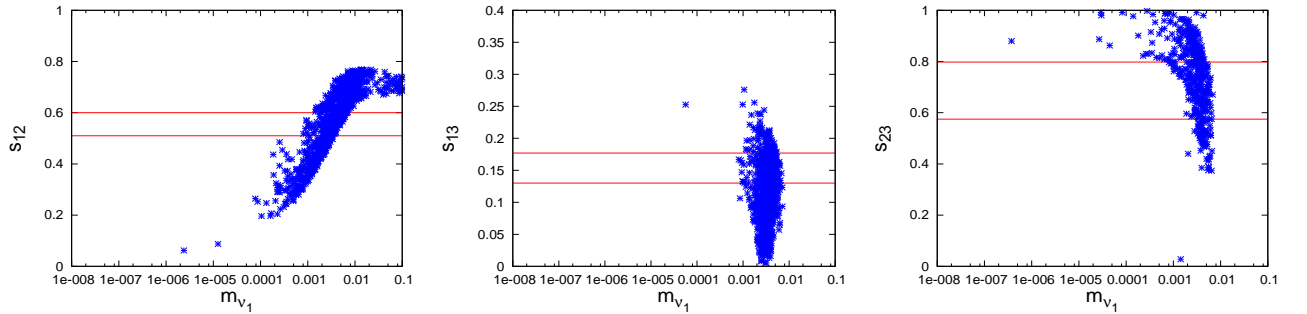


Figure 9: Plots showing the dependence of mixing angles on the lightest neutrino mass when the other two angles are constrained by their 3σ ranges for Class I ansatz of texture four zero Dirac mass matrices (normal hierarchy) .

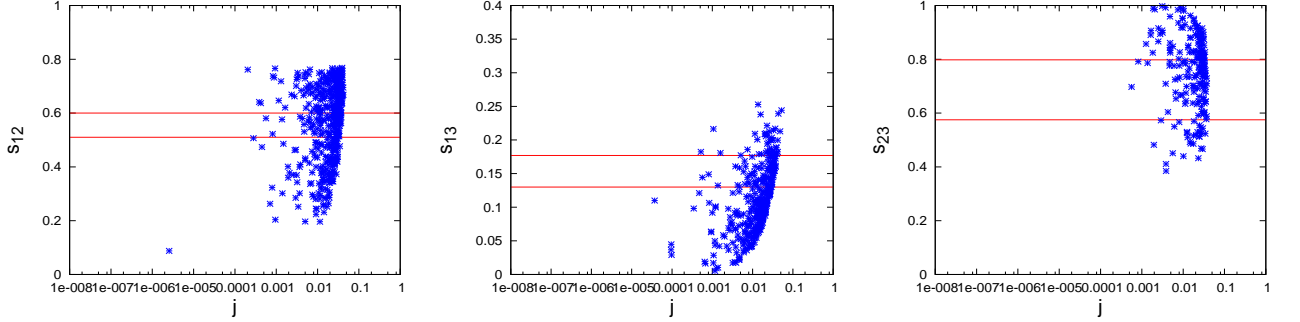


Figure 10: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for Class I ansatz of texture four zero Dirac mass matrices (normal hierarchy).

$m_{\nu 1} \lesssim m_{\nu 2} \sim m_{\nu 3} \sim 0.1\text{eV}$ or $m_{\nu 3} \sim m_{\nu 1} \lesssim m_{\nu 2} \sim 0.1\text{eV}$, corresponding to normal hierarchy and inverted hierarchy respectively. Since while carrying out the calculations pertaining to both the normal as well as inverted hierarchy cases, the range of lightest neutrino mass is taken to be $10^{-8} - 10^{-1}$ eV, which includes the neutrino masses corresponding to the degenerate scenario; therefore, by discussion similar to the one given for ruling out inverted hierarchy, class I ansatz seems to be ruled out for degenerate scenario of neutrino masses as well. Similarly, from figure (9) the value of the lightest neutrino mass pertaining to the degenerate scenario, $m_{\nu 1} \sim 0.1\text{eV}$, seems to be outside the experimentally allowed region, thereby ruling out Class I ansatz for both the cases of degenerate scenario of neutrino masses.

4.2.2 Class II ansatz

To analyse this class we follow the same procedure as for class I ansatz. The charged lepton and neutrino mass matrices which we choose to analyse for this class can be given as,

$$M_i = \begin{pmatrix} D_i & A_i e^{i\alpha_i} & 0 \\ A_i e^{-i\alpha_i} & 0 & B_i e^{i\beta_i} \\ 0 & B_i e^{-i\beta_i} & E_i \end{pmatrix}, \quad (17)$$

where $i = l, \nu_D$ corresponds to the charged lepton and Dirac neutrino mass matrices respectively. As for class I, the elements D_l and D_ν as well as the phases ϕ_1 and ϕ_2 are considered to be the free parameters. Firstly, we examine the viability of

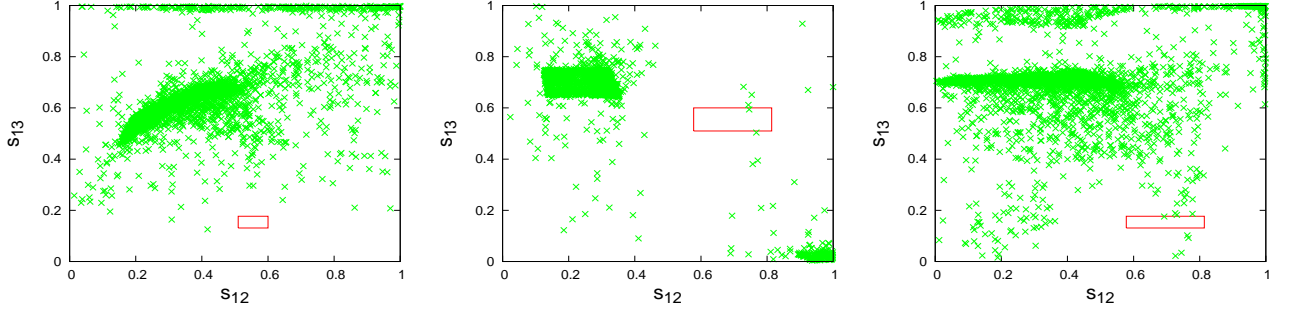


Figure 11: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class II ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

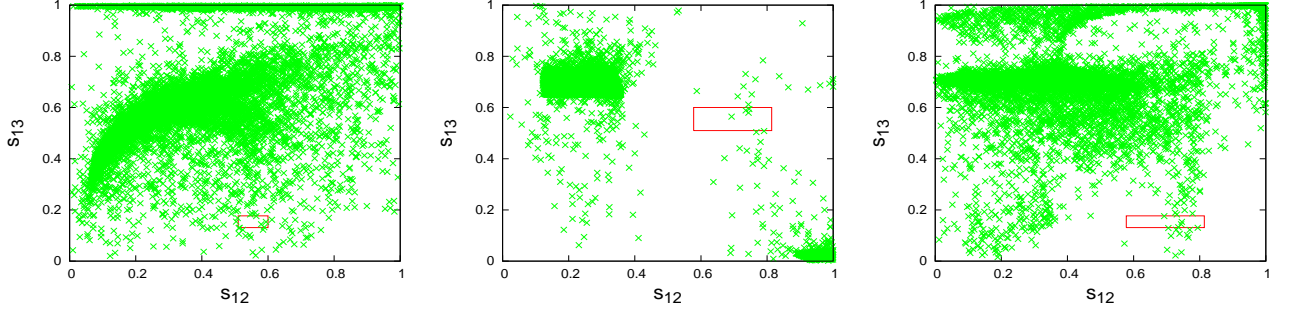


Figure 12: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range for Class II ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

inverted hierarchy for the structure given in eqn.(17). To this end in figures (11) and (12), we present the plots showing the parameter space for two mixing angles wherein the third angle is constrained by its 1σ range and 3σ range respectively. The rectangular boxes in these plots show the 3σ ranges for the two mixing angles being considered. It is interesting to note that the inverted hierarchy seems to be ruled at by the 1σ ranges for the present mixing data, while it seems to be compatible with the 3σ experimental bounds.

Next, in order to examine the compatibility of structure given in eqn.(17) with the normal hierarchy, in figure (13) we present the plots showing the parameter space for two mixing angles wherein the third angle is constrained by its 1σ experimental range. A general look at figure (13) reveals that normal hierarchy is compatible with the structure (17) at 1σ level.

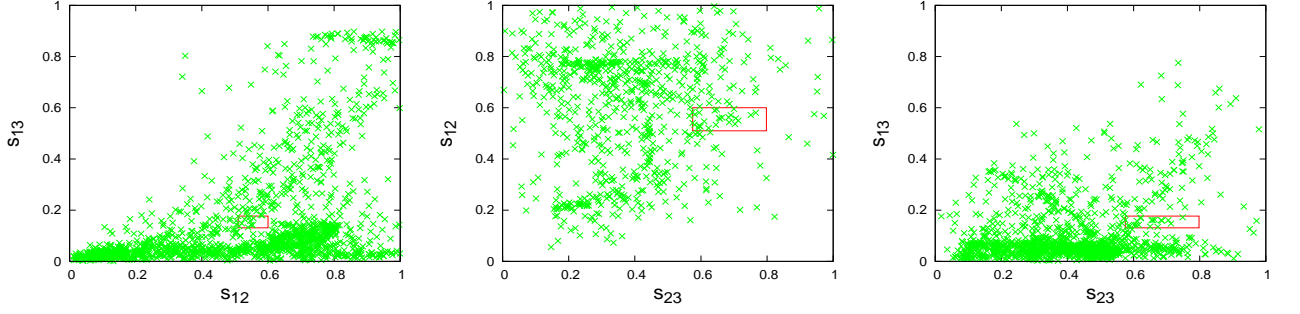


Figure 13: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class II ansatz of texture four zero Dirac mass matrices (normal hierarchy).

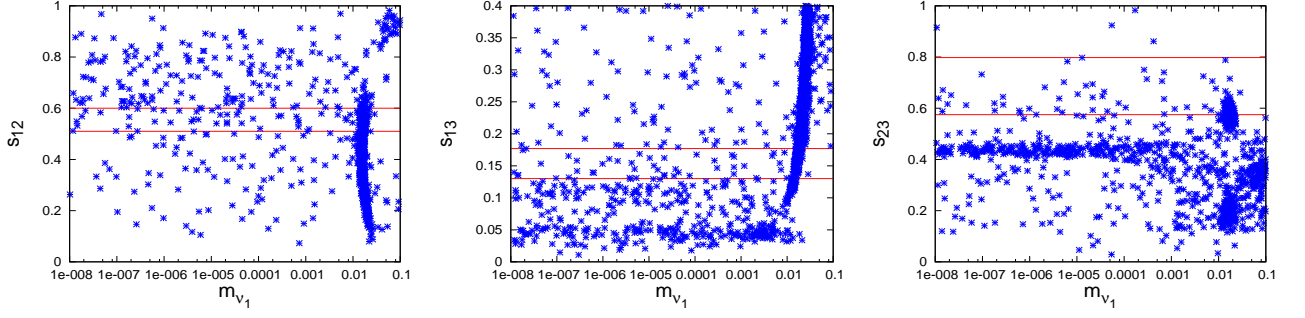


Figure 14: Plots showing the dependence of mixing angles on the lightest neutrino mass when the other two angles are constrained by their 3σ ranges for Class II ansatz of texture four zero Dirac mass matrices (normal hierarchy).

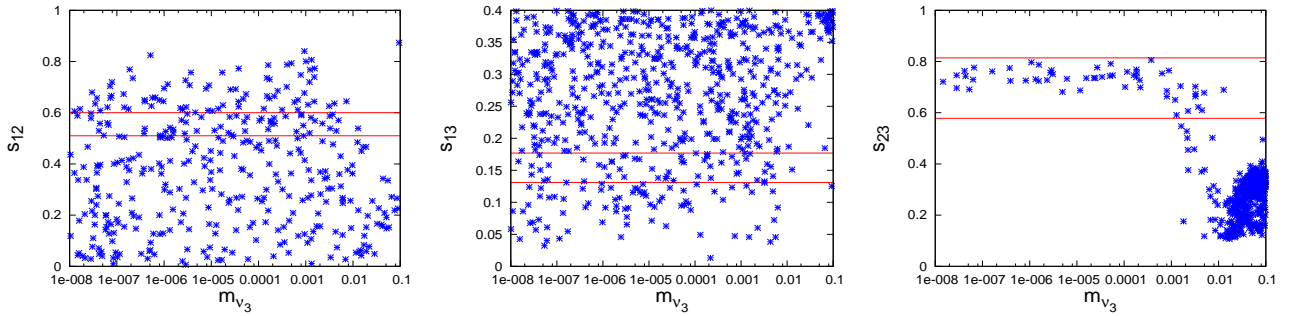


Figure 15: Plots showing the dependence of mixing angles on the lightest neutrino mass when the other two angles are constrained by their 3σ ranges for Class II ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

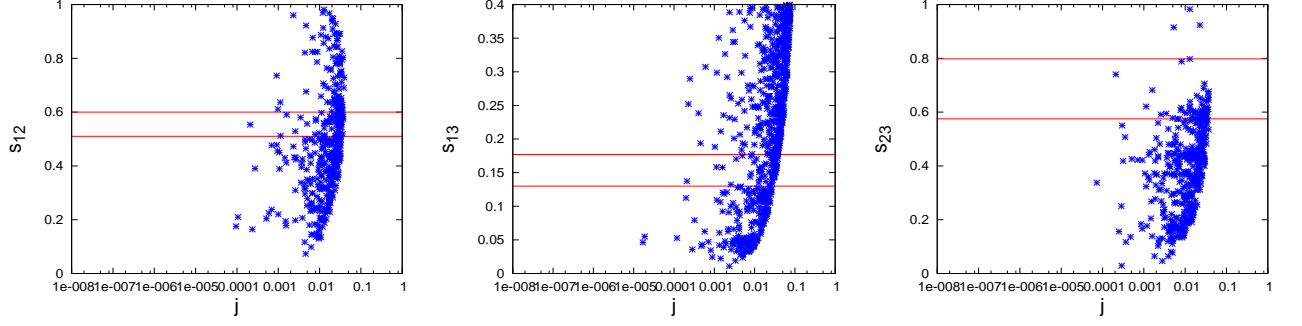


Figure 16: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for Class II ansatz of texture four zero Dirac mass matrices (normal hierarchy).

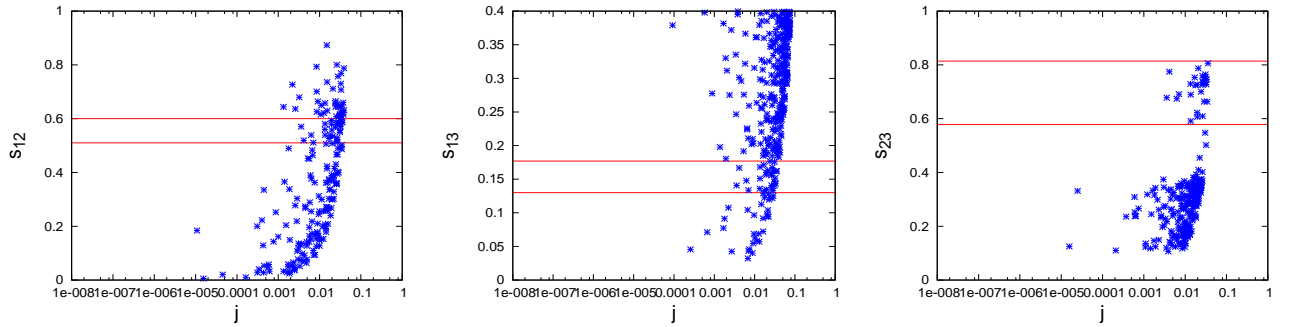


Figure 17: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for Class II ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

Thus, we find that both the normal as well as inverted neutrino mass hierarchies are compatible with the 3σ ranges for the lepton mixing data. As a next step, in figures (14) and (15) we attempt to study the implications of the 3σ ranges of the leptonic mixing angles on the lightest neutrino mass for this class for the normal and inverted neutrino mass hierarchy respectively. As can be seen from these figures, for both the neutrino mass hierarchies the 3σ ranges of the mixing angles provide only an upper bound on the lightest neutrino mass, viz. $m_{\nu 1} \lesssim 0.01eV$ and $m_{\nu 3} \lesssim 0.01eV$ for the normal and inverted hierarchy respectively. This bound seems to rule out both the degenerate neutrino mass ordering scenarios as the bound on the lightest neutrino mass does not include the value of the lightest neutrino mass pertaining to the degenerate scenario ($\sim 0.1eV$).

Further, we study the possibility of CP violation in the leptonic sector for this class by studying the variation of Jarlskog's rephasing invariant J with all the mixing angles for normal as well as inverted neutrino mass hierarchy in figures (16) and (17) respectively. While plotting these graphs the two mixing angles, other than the one being considered, are constrained by their 3σ ranges. The parallel lines in these plots show the 3σ experimental ranges for the mixing angle being considered. It is interesting to note that for the case of normal hierarchy one gets a broader range for J as compared to the one for the case of inverted hierarchy. To be more explicit, one finds $0.0001 \lesssim J \lesssim 0.05$ and $0.003 \lesssim J \lesssim 0.02$ corresponding to normal and inverted hierarchy respectively.

4.2.3 Class III ansatz

To study the lepton mass matrices for this class, we choose to analyse the following structure,

$$M_i = \begin{pmatrix} 0 & A_i e^{i\alpha_i} & B_i e^{i\gamma_i} \\ A_i e^{-i\alpha_i} & 0 & D_i e^{i\beta_i} \\ B_i e^{-i\gamma_i} & D_i e^{-i\beta_i} & E_i \end{pmatrix}, \quad (18)$$

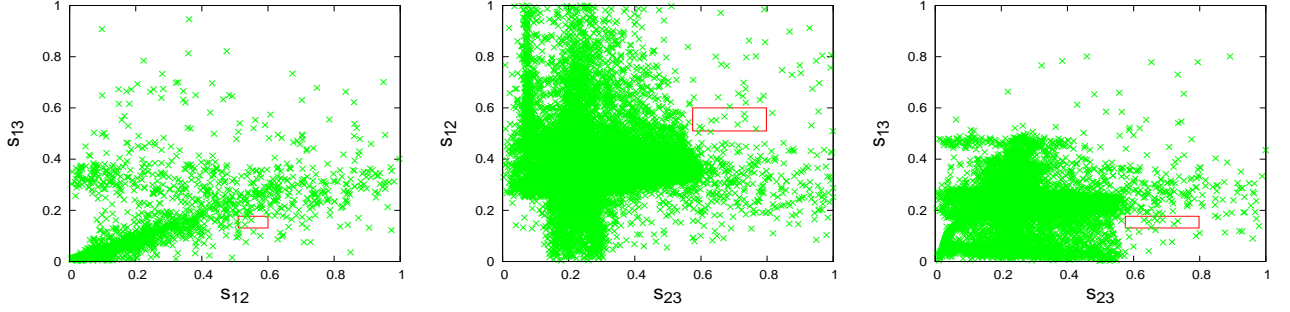


Figure 18: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class III ansatz of texture four zero Dirac mass matrices (normal hierarchy).

where $i = l$, ν_D corresponds to the charged lepton and Dirac neutrino mass matrices respectively. In the case of factorizable phases, the lepton mass matrices belonging to this class can be analysed following a methodology similar to that for class I and class II ansatz. For the purpose of calculations, the (2,3) element in each sector, D_l and D_ν , as well as the phases ϕ_1 and ϕ_2 are considered to be the free parameters. It is interesting to note that in this class as well both the normal as well as inverted mass neutrino mass orderings seem to be viable as can be seen from figures (18) and (19).

Further, in figures (20) and (21) we attempt to study the dependence of the leptonic mixing angles on the lightest neutrino mass for this class for the normal and inverted neutrino mass hierarchy respectively. As can be seen from these figures, the lightest neutrino mass seems to be unrestricted by the 3σ ranges of the mixing angles for both the neutrino mass hierarchies. Since the value of the lightest neutrino mass pertaining to the degenerate scenario ($\sim 0.1\text{eV}$) is included in the range allowed by the 3σ ranges of the mixing angles, therefore both the degenerate scenarios may be viable for this class of lepton mass matrices.

Further, we study the possibility of CP violation in the leptonic sector for this class by studying the variation of Jarlskog's rephasing invariant J with all the mixing angles for normal as well as inverted neutrino mass hierarchy in figures (22) and (23) respectively. While plotting these graphs the two mixing angles, other than the one being considered, are constrained by their 3σ ranges. The parallel lines in these

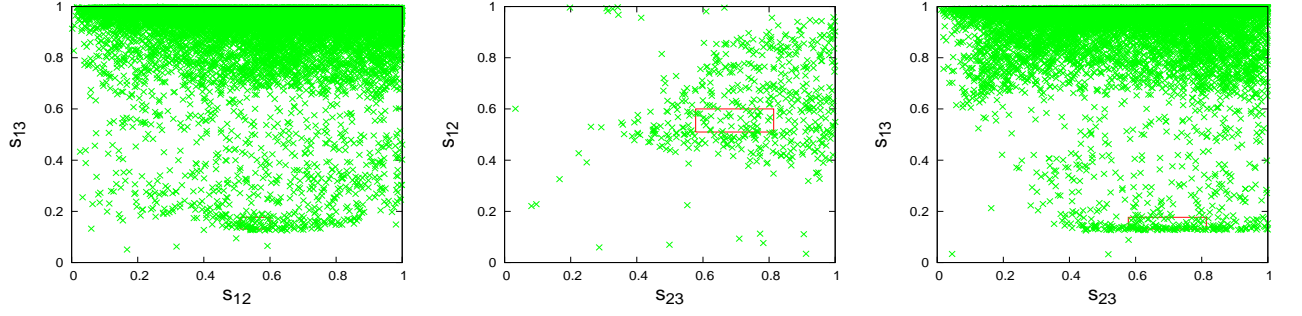


Figure 19: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class III ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

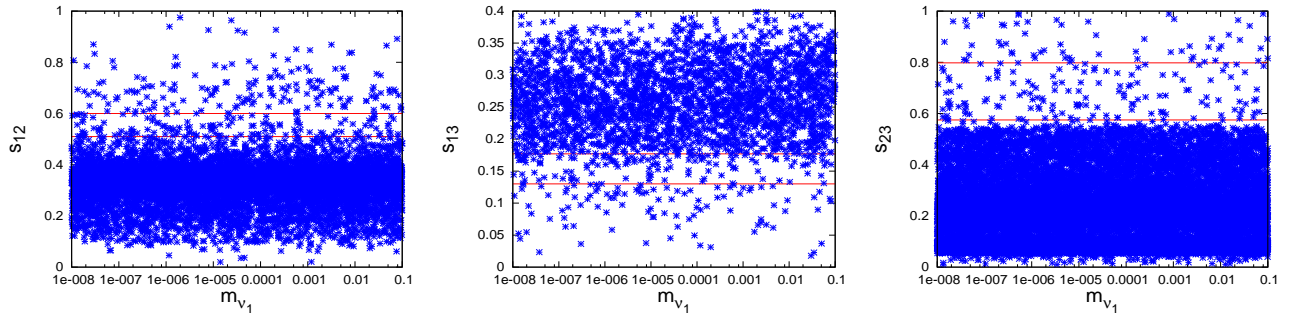


Figure 20: Plots showing the dependence of mixing angles on the lightest neutrino mass when the other two angles are constrained by their 3σ ranges for Class III ansatz of texture four zero Dirac mass matrices (normal hierarchy).

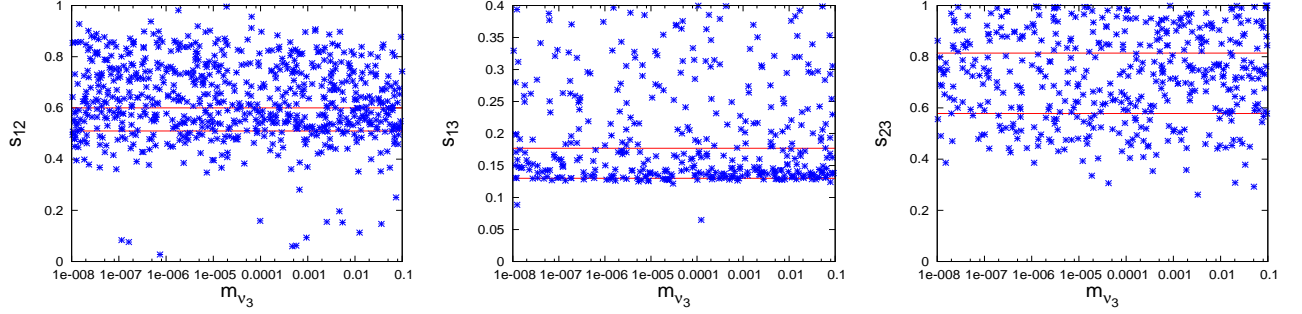


Figure 21: Plots showing the dependence of mixing angles on the lightest neutrino mass when the other two angles are constrained by their 3σ ranges for Class III ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

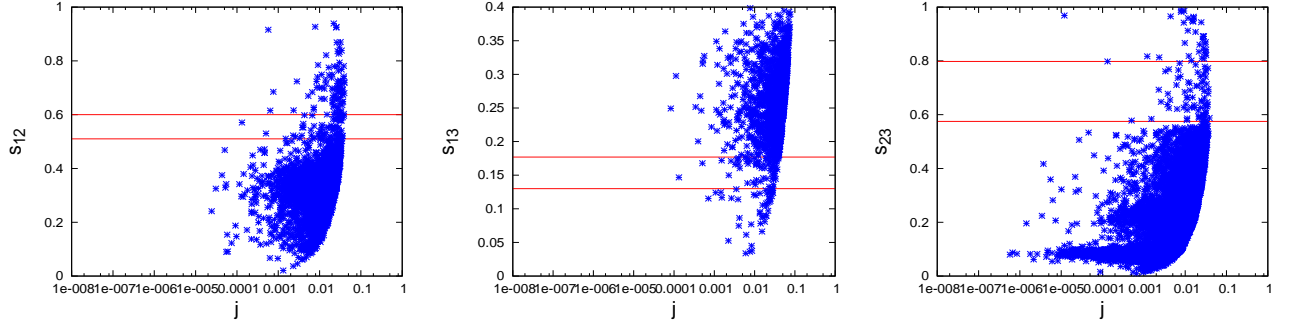


Figure 22: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for Class III ansatz of texture four zero Dirac mass matrices (normal hierarchy).

plots show the 3σ experimental ranges for the mixing angle being considered. It is interesting to note that for the case of inverted hierarchy one gets a broader range for J as compared to the one for the case of normal hierarchy, contrary to the observation in class II ansatz. To be more explicit, one finds $0.0001 \lesssim J \lesssim 0.05$ and $0.00001 \lesssim J \lesssim 0.1$ corresponding to normal and inverted hierarchy scenario respectively.

4.3 Texture five zero lepton mass matrices

After studying all possible texture four zero lepton mass matrices, it becomes interesting to explore the parallel texture five zero structures for each class which can be derived by substituting either $D_l = 0$, $D_\nu \neq 0$ or $D_l \neq 0$, $D_l = 0$ in the correspond-

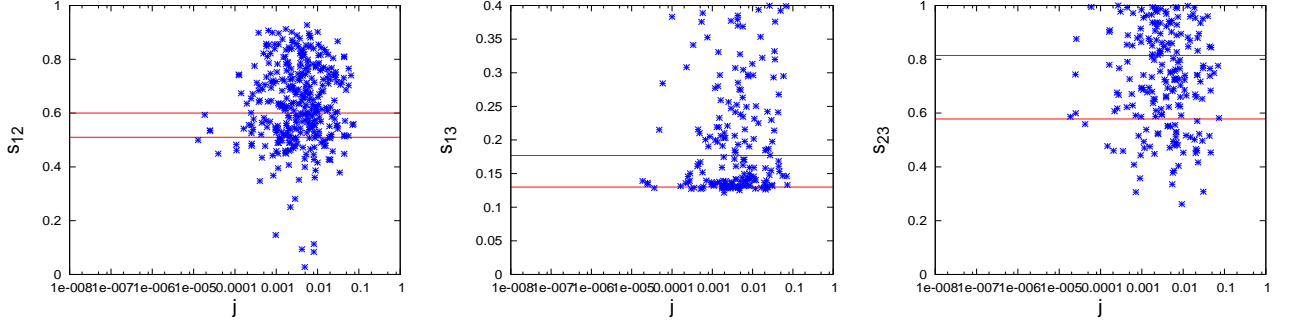


Figure 23: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for Class III ansatz of texture four zero Dirac mass matrices (inverted hierarchy).

ing texture four zero mass matrices. In this subsection, we carry out a detailed study of all classes of texture five zero lepton mass matrices for Dirac neutrinos pertaining to both the possibilities leading to texture five zero structures.

4.3.1 Class I ansatz

The two possibilities for texture five zero lepton mass matrices for this class can be given as,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \quad (19)$$

or

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}. \quad (20)$$

We study both these possibilities in detail for all the neutrino mass orderings. Firstly, we examine the compatibility of matrices (19) and (20) with the inverted hierarchy of neutrino masses. For this purpose, in figures (24) and (25), we present the plots showing the parameter space allowed by this ansatz for any two mixing angles wherein the third one is constrained by its 3σ experimental bound for inverted hierarchy of neutrino masses. The rectangular regions in these plots represent the 3σ ranges for the two mixing angles being considered. A general look at these plots

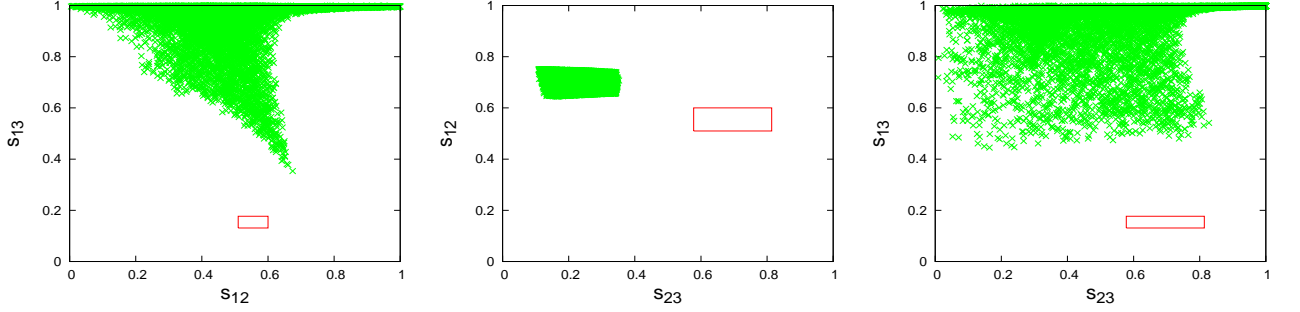


Figure 24: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class I ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

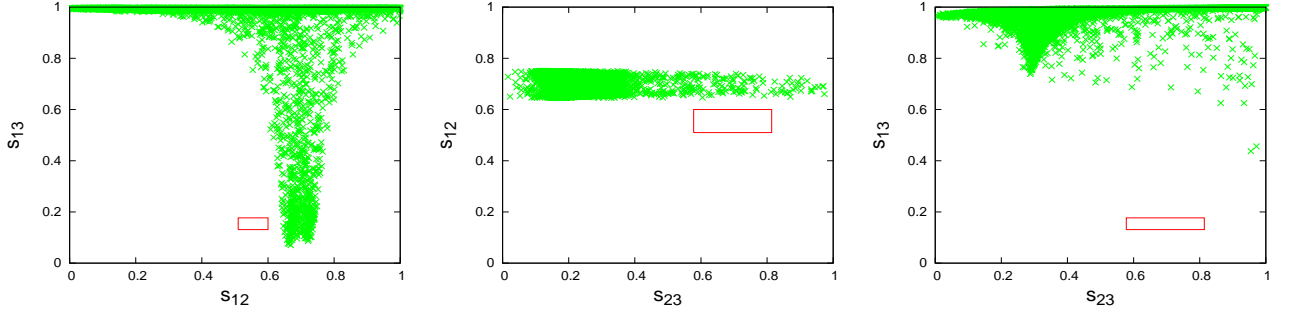


Figure 25: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class I ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

shows that inverted hierarchy is ruled out for both the texture five zero possibilities for this class.

After ruling out the structures (19) and (20) for the inverted hierarchy, we now proceed to examine the compatibility of these matrices for the normal hierarchy case. To this end, in figures (26) and (27), we present the plots showing the parameter space corresponding to any two mixing angles wherein the third one is constrained by its 3σ range. Interestingly, normal hierarchy seems to be ruled out for the case $D_\nu = 0$ and $D_l \neq 0$, whereas for the case $D_\nu \neq 0$ and $D_l = 0$ it seems to be viable as can be seen from the figure (26), wherein the parameter space shows significant overlap with the experimentally allowed 3σ region shown by the rectangular boxes in each plot. For the $D_l = 0$ and $D_\nu \neq 0$ case for texture five zero mass matrices, wherein normal hierarchy has been shown to be viable, we proceed to study the dependence of the

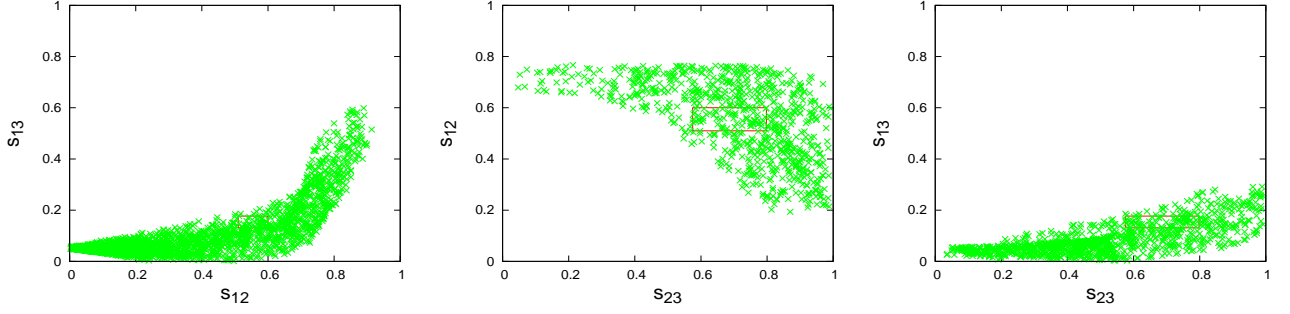


Figure 26: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class I ansatz of texture five zero Dirac mass matrices (normal hierarchy).

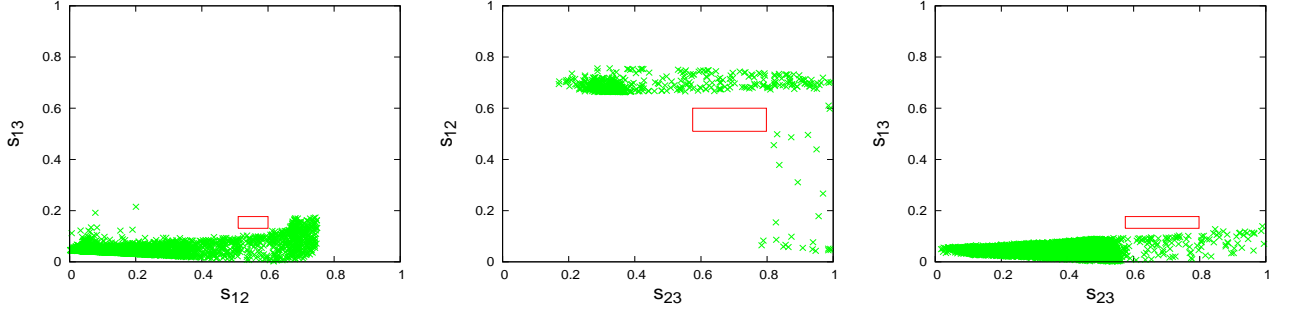


Figure 27: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class I ansatz of texture five zero Dirac mass matrices (normal hierarchy).

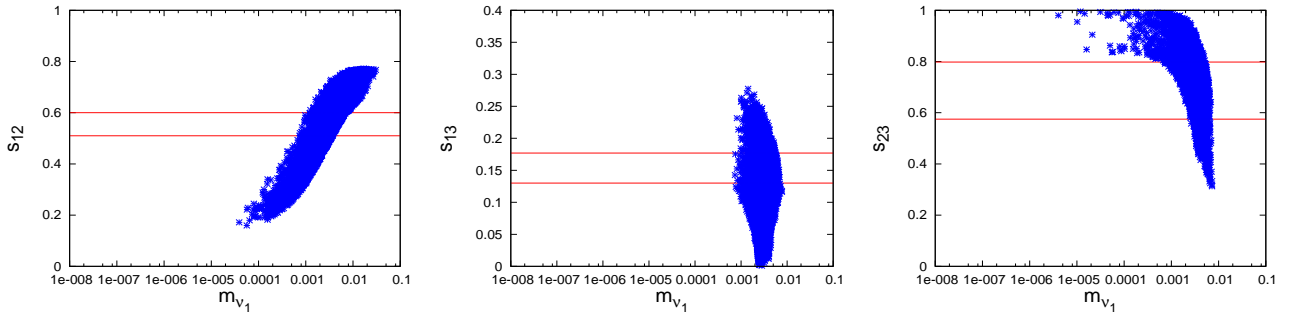


Figure 28: Plots showing the lightest neutrino mass with mixing angles when the other two angles are constrained by their 3σ ranges $D_l = 0$ and $D_\nu \neq 0$ scenario for Class I ansatz of texture five zero Dirac mass matrices (normal hierarchy).

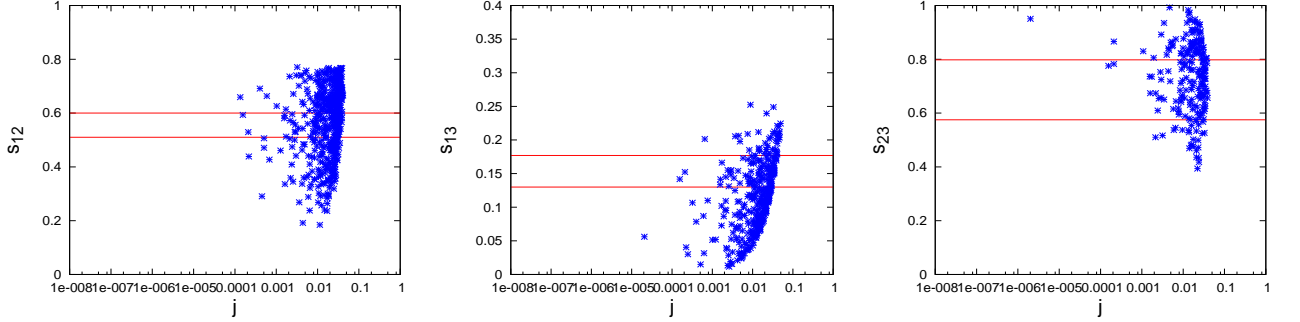


Figure 29: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges $D_l = 0$ and $D_\nu \neq 0$ scenario for Class I ansatz of texture five zero Dirac mass matrices (normal hierarchy).

lightest neutrino mass and Jarlskog's parameter on the the leptonic mixing angles. To this end, we present the plots showing variation of the lightest neutrino mass and Jarlskog's parameter with the mixing angles in figures (28) and (29) respectively. While plotting these graphs, the other two mixing angles have been constrained by their 3σ ranges. The parallel lines in these plots show the 3σ experimental ranges for the mixing angle being considered. Having a careful look at these plots, one can find the ranges for the lightest neutrino mass and the Jarlskog's parameter for this case of texture five zero matrices, viz., $0.001eV \lesssim m_{\nu 1} \lesssim 0.01eV$, $0.00001 \lesssim j \lesssim 0.05$. Further, since inverted hierarchy is ruled out for both the cases for texture five zero and for the normal hierarchy the range of the lightest neutrino mass does not include that for the degenerate neutrino mass ordering, therefore both the cases for degenerate scenario seems to be ruled out for this class.

4.3.2 Class II ansatz

The two possibilities for texture five zero lepton mass matrices for this class can be given as,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} D_\nu & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \quad (21)$$

or

$$M_l = \begin{pmatrix} D_l & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \quad (22)$$

We study both these possibilities in detail for all the neutrino mass orderings. Firstly, we examine the compatibility of matrices (21) and (22) with the inverted hierarchy of neutrino masses. For this purpose, in figures (30) and (31), we present the plots showing the parameter space allowed by this ansatz for any two mixing angles wherein the third one is constrained by its 3σ experimental bound for inverted hierarchy of neutrino masses. The rectangular regions in these plots represent the 3σ ranges for the two mixing angles being considered. Interestingly, one finds that for the case $D_l = 0$ and $D_\nu \neq 0$ of texture five zero lepton mass matrices inverted hierarchy is ruled out, whereas for the case $D_l \neq 0$ and $D_\nu = 0$ of texture five zero lepton mass matrices inverted hierarchy scenario seems to be viable.

For the $D_l \neq 0$ and $D_\nu = 0$ case of lepton mass matrices, wherein inverted hierarchy is shown to be viable, we proceed next to study the dependence of the lightest neutrino mass and Jarlskog's parameter on the leptonic mixing angles. To this end, we present the plots showing variation of the lightest neutrino mass and Jarlskog's parameter with the mixing angles in figures (32) and (33) respectively. While plotting these graphs, the other two mixing angles have been constrained by their 3σ ranges. Interestingly, for this case one finds a very narrow range for both the lightest neutrino mass as well the Jarlskog's parameter as can be seen from figures (32) and (33).

After studying both the cases for texture five zero mass matrices pertaining to class II ansatz for inverted hierarchy, we now carry out a similar analysis pertaining to normal hierarchy. To this end, in figures (34) and (35), we present the plots showing the parameter space corresponding to any two mixing angles wherein the third one is constrained by its 3σ range. Interestingly, normal hierarchy seems to be viable for both the cases, $D_\nu = 0$ and $D_l \neq 0$ as well as $D_\nu \neq 0$ and $D_l = 0$, of texture five zero lepton mass matrices as can be seen from these plots (34) and (35), wherein

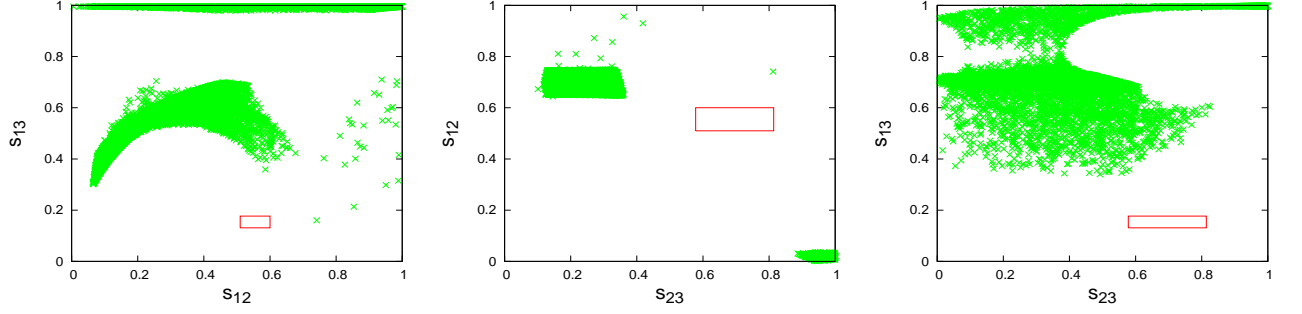


Figure 30: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

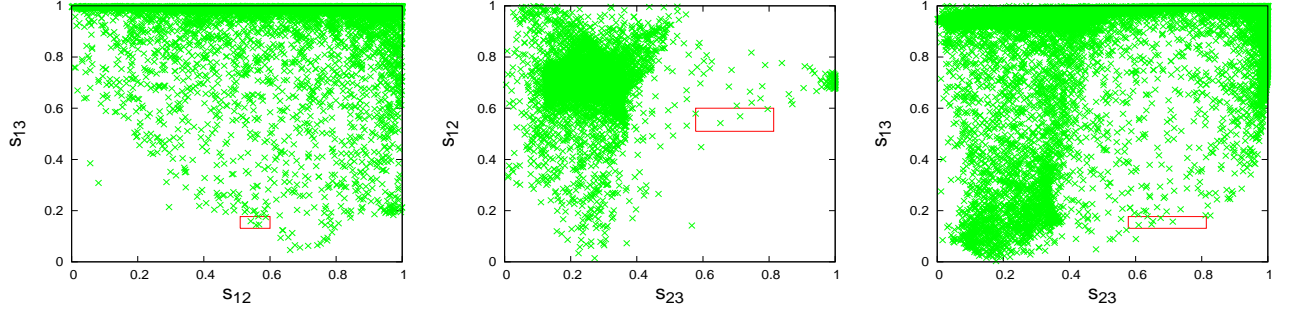


Figure 31: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

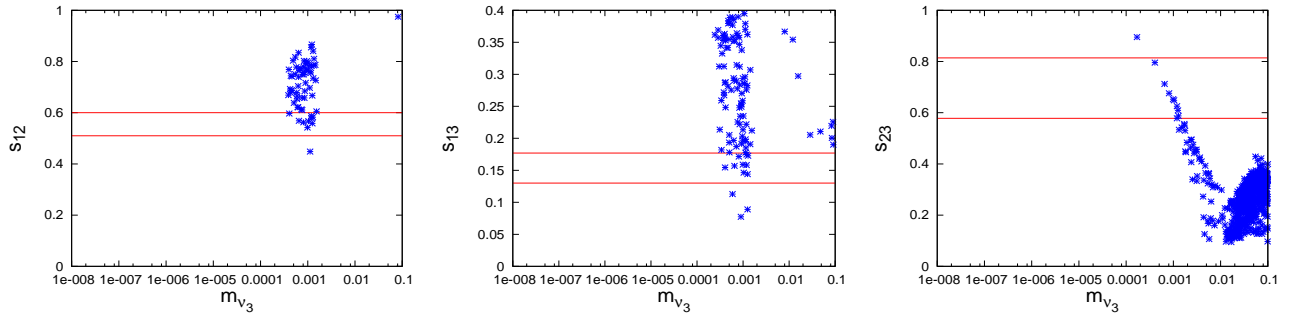


Figure 32: Plots showing the lightest neutrino mass with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l \neq 0$ and $D_\nu = 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

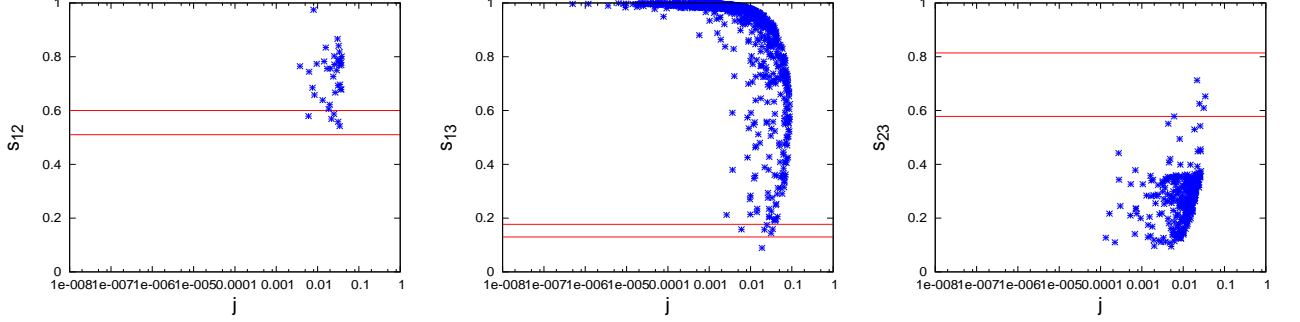


Figure 33: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l \neq 0$ and $D_\nu = 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

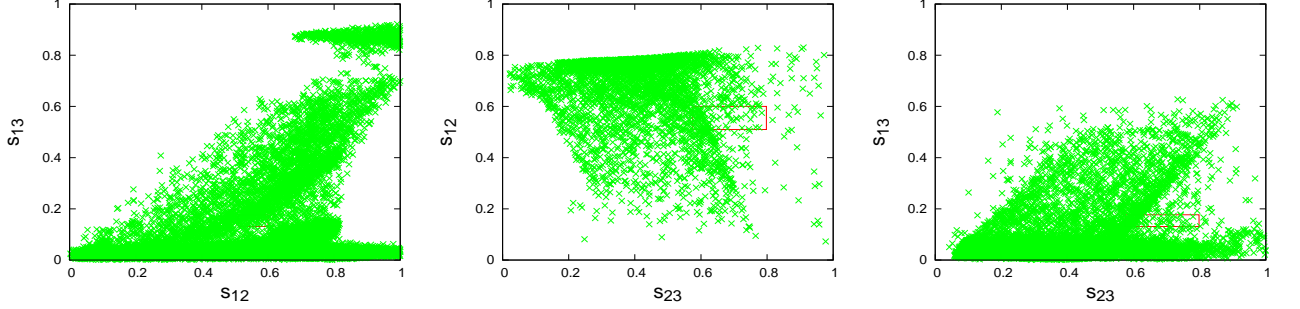


Figure 34: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (normal hierarchy).

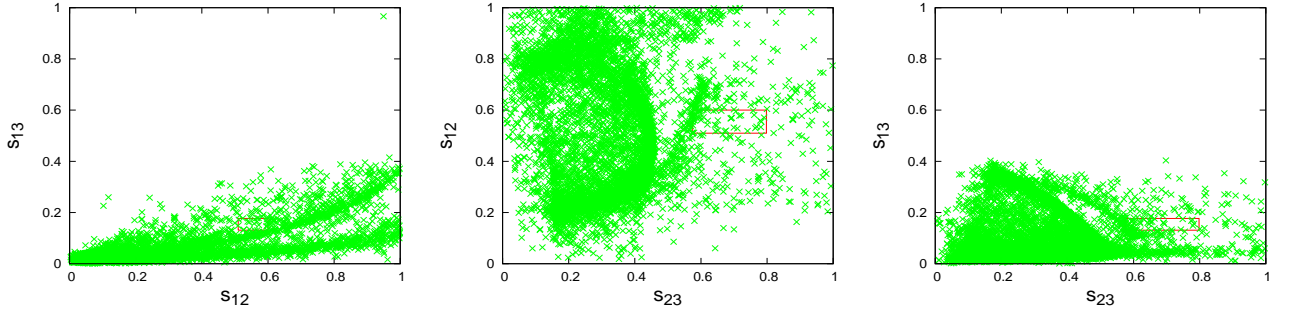


Figure 35: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (normal hierarchy).

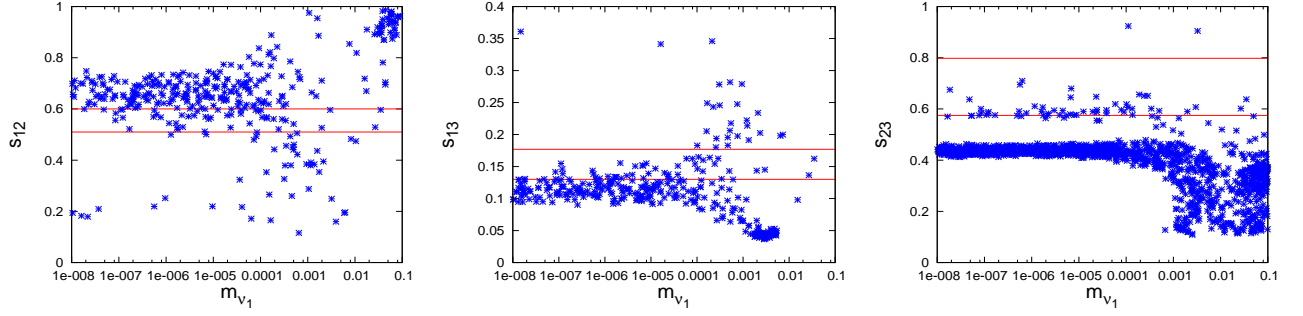


Figure 36: Plots showing the lightest neutrino mass with mixing angles when the other two angles are constrained by their 1σ ranges for $D_l \neq 0$ and $D_\nu = 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (normal hierarchy).

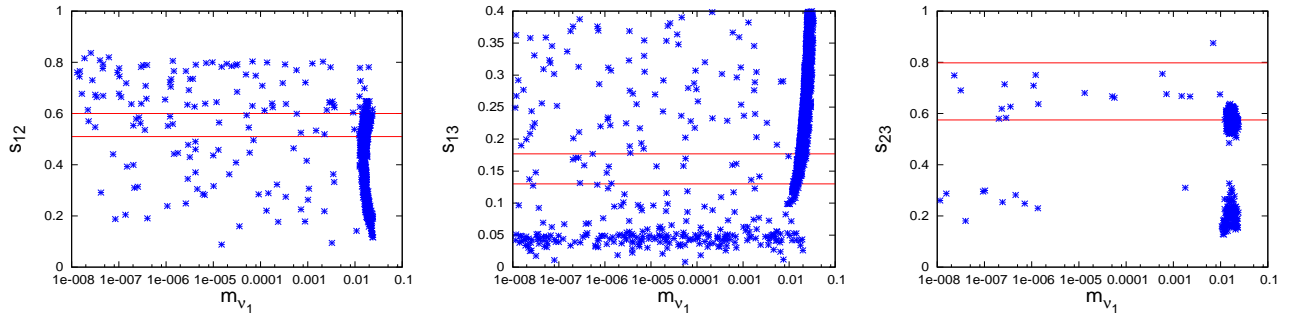


Figure 37: Plots showing the lightest neutrino mass with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l = 0$ and $D_\nu \neq 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (normal hierarchy).

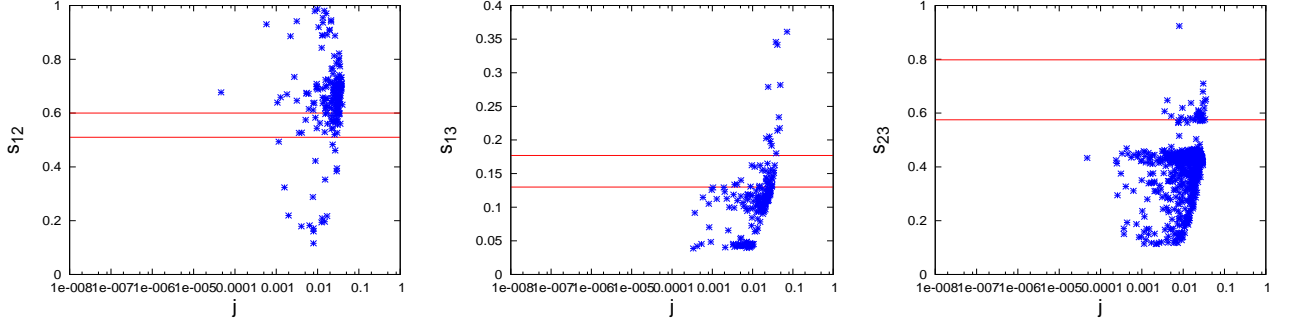


Figure 38: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l \neq 0$ and $D_\nu = 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (normal hierarchy).

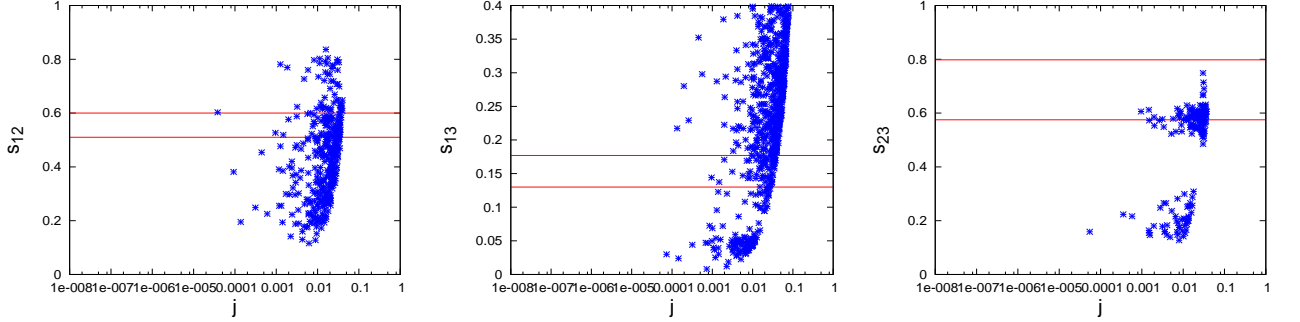


Figure 39: Plots showing the variation of Jarlskog CP violating parameter with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l = 0$ and $D_\nu \neq 0$ scenario for Class II ansatz of texture five zero Dirac mass matrices (normal hierarchy).

the parameter space shows significant overlap with the experimentally allowed 3σ region shown by the rectangular boxes in each plot. Next, we study the dependence of the lightest neutrino mass and Jarlskog's parameter on the the leptonic mixing angles for this case. To this end, we present the plots showing variation of the lightest neutrino mass with the mixing angles in figures (36) and (37) respectively. While plotting these graphs, the other two mixing angles have been constrained by their 3σ ranges. Interestingly, for $D_l \neq 0$ and $D_\nu = 0$, the lightest neutrino mass is largely unrestricted, whereas for the case $D_l = 0$ and $D_\nu \neq 0$ one obtains an upper bound $\approx 0.01eV$ for the lightest neutrino mass.

Further, in figures (38) and (39) we present the graphs showing the variation

of the Jarlskog's parameter with each of the mixing angles, keeping the other two constrained by their 3σ ranges. Parallel lines in these graphs show the 3σ ranges for the mixing angles being considered. Interestingly, one finds that the range of J for $D_l \neq 0$ and $D_\nu = 0$ is quite narrow as compared to the $D_l = 0$ and $D_\nu \neq 0$ case of texture five zero lepton mass matrices for class II ansatz.

4.3.3 Class III ansatz

The two possibilities for texture five zero lepton mass matrices for this class can be given as,

$$M_l = \begin{pmatrix} 0 & A_l e^{i\alpha_l} & B_l \\ A_l e^{-i\alpha_l} & 0 & 0 \\ B_l & 0 & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu e^{i\alpha_\nu} & B_\nu \\ A_\nu e^{-i\alpha_\nu} & 0 & D_\nu e^{i\beta_\nu} \\ B_\nu & D_\nu e^{-i\beta_\nu} & E_\nu \end{pmatrix}, \quad (23)$$

or

$$M_l = \begin{pmatrix} 0 & A_l e^{i\alpha_l} & B_l \\ A_l e^{-i\alpha_l} & 0 & D_l e^{i\beta_l} \\ B_l & D_l e^{-i\beta_l} & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu e^{i\alpha_\nu} & B_\nu \\ A_\nu e^{-i\alpha_\nu} & 0 & 0 \\ B_\nu & 0 & E_\nu \end{pmatrix}, \quad (24)$$

We study both these possibilities in detail for all the neutrino mass orderings. Firstly, we examine the compatibility of matrices (23) and (24) with the inverted hierarchy of neutrino masses. For this purpose, in figures (40) and (41), we present the plots showing the parameter space allowed by this ansatz for any two mixing angles wherein the third one is constrained by its 3σ experimental bound for inverted hierarchy of neutrino masses. The rectangular regions in these plots represent the 3σ ranges for the two mixing angles being considered. Interestingly, one finds that for the case $D_l = 0$ and $D_\nu \neq 0$ of texture five zero lepton mass matrices inverted hierarchy is ruled out, whereas for the case $D_l \neq 0$ and $D_\nu = 0$ of texture five zero lepton mass matrices inverted hierarchy scenario seems to be viable.

For the $D_l \neq 0$ and $D_\nu = 0$ case of lepton mass matrices, wherein inverted

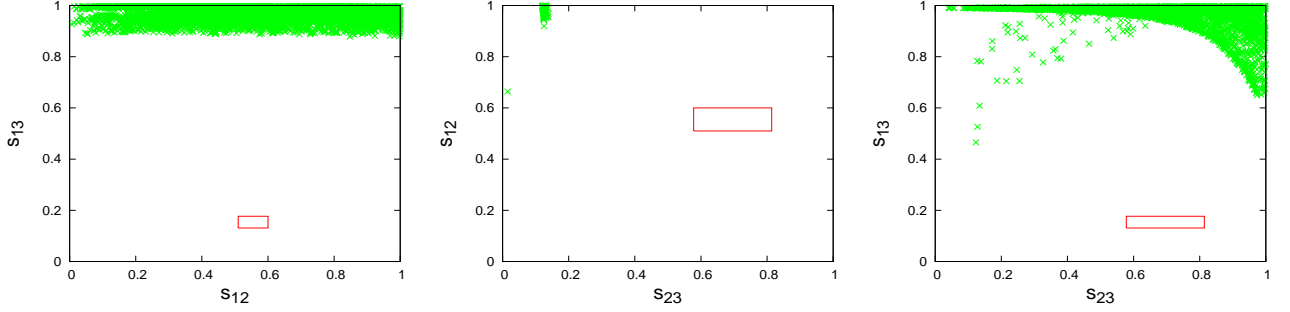


Figure 40: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class III ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

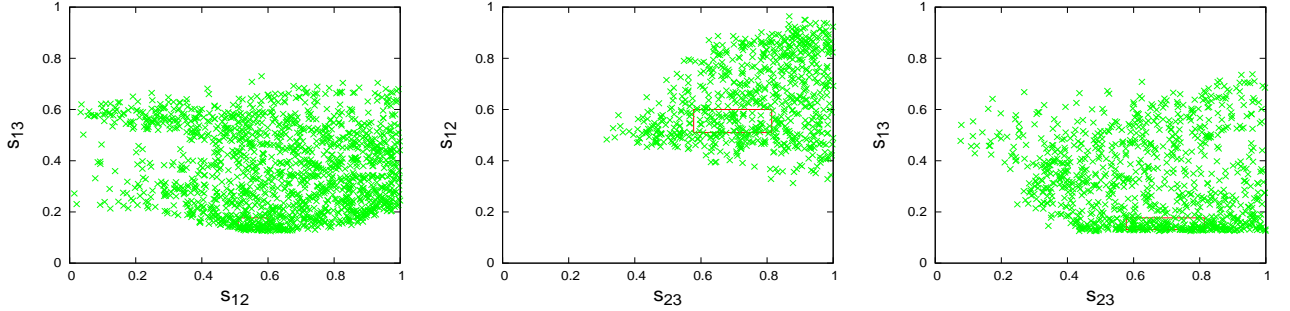


Figure 41: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class III ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

hierarchy is shown to be viable, we proceed next to study the dependence of the lightest neutrino mass and Jarlskog's parameter on the leptonic mixing angles. To this end, we present the plots showing variation of the lightest neutrino mass with the mixing angles in figures (42). While plotting these graphs, the other two mixing angles have been constrained by their 3σ ranges. Interestingly, one finds that the lightest neutrino mass is unrestricted for this structure. Further, on carrying out the calculations for the Jarlskog's parameter, one finds that the 3σ experimental bounds for the mixing angles allow an extremely narrow range ~ 0.05 for Jarlskog's parameter.

After studying both the cases for texture five zero mass matrices for inverted hierarchy pertaining to class II ansatz, we now carry out a similar analysis pertaining to normal hierarchy. To this end, in figures (43) and (44), we present the plots

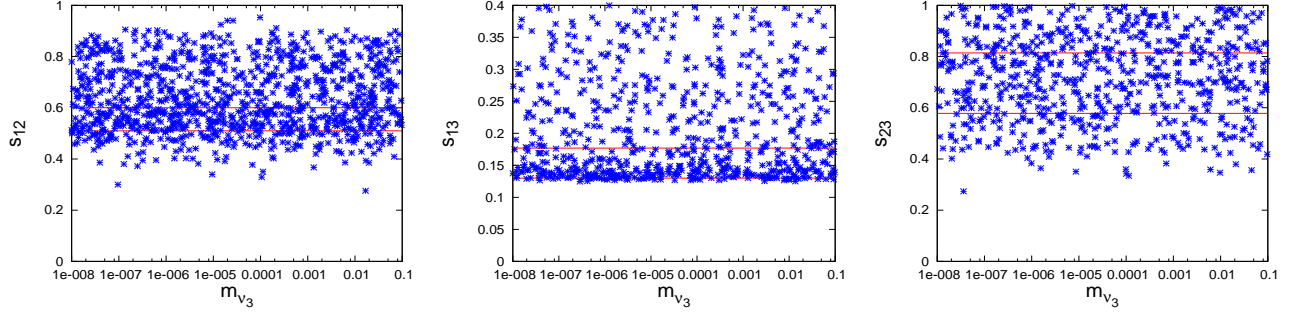


Figure 42: Plots showing the lightest neutrino mass with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l \neq 0$ and $D_\nu = 0$ scenario for Class III ansatz of texture five zero Dirac mass matrices (inverted hierarchy).

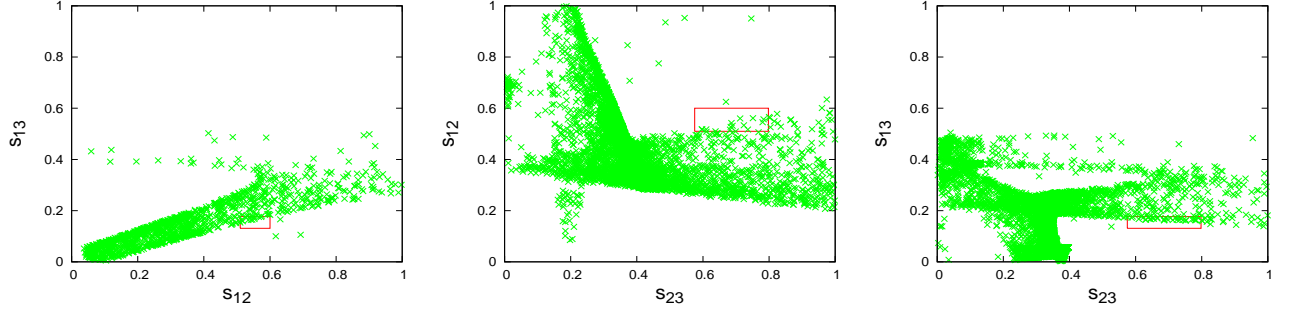


Figure 43: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class III ansatz of texture five zero Dirac mass matrices (normal hierarchy).

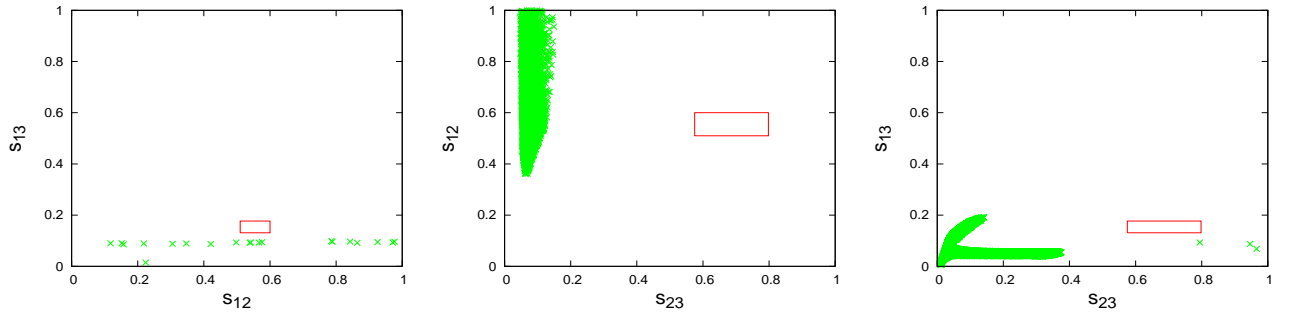


Figure 44: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class III ansatz of texture five zero Dirac mass matrices (normal hierarchy).

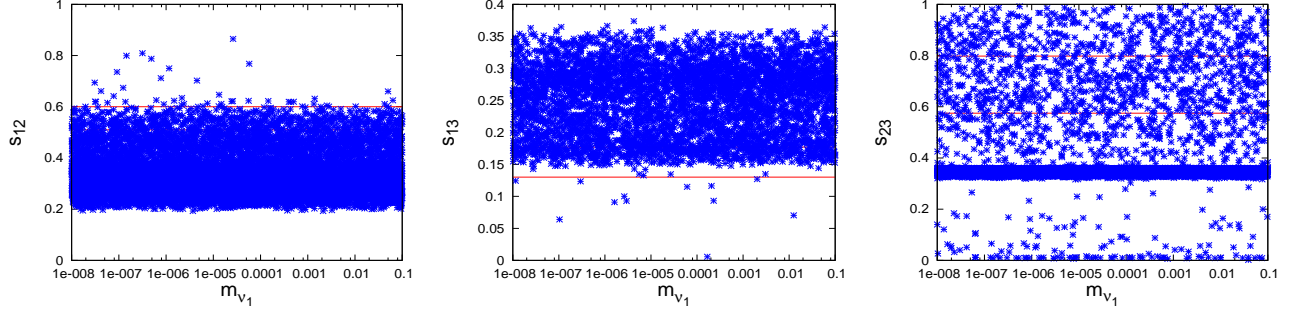


Figure 45: Plots showing the variation of lightest neutrino mass with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l = 0$ and $D_\nu \neq 0$ scenario for Class III ansatz of texture five zero Dirac mass matrices (normal hierarchy).

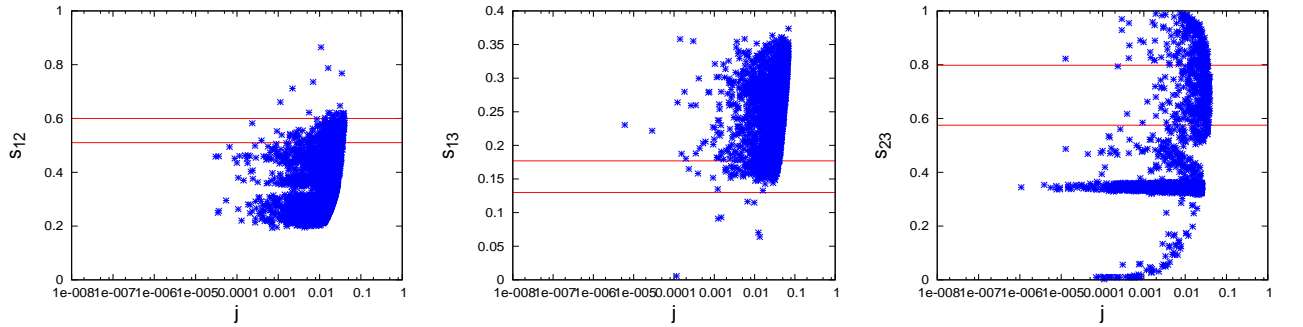


Figure 46: Plots showing the variation of Jarlskog's parameter with mixing angles when the other two angles are constrained by their 3σ ranges for $D_l = 0$ and $D_\nu \neq 0$ scenario for Class III ansatz of texture five zero Dirac mass matrices (normal hierarchy).

showing the parameter space corresponding to any two mixing angles wherein the third one is constrained by its 3σ range. Interestingly, normal hierarchy seems to be ruled out for the case $D_l \neq 0$ and $D_\nu = 0$, whereas for the case $D_l = 0$ and $D_\nu \neq 0$ of texture five zero lepton mass matrices normal hierarchy seems to be viable. Next, we study the dependence of the lightest neutrino mass and Jarlskog's parameter on the leptonic mixing angles for the $D_l = 0$ and $D_\nu \neq 0$ case of texture five zero mass matrices corresponding to class III. To this end, we present the plots showing variation of the lightest neutrino mass and Jarlskog's parameter with the mixing angles in figures (45) and (46) respectively. While plotting these graphs, the other two mixing angles have been constrained by their 3σ ranges. Interestingly, one finds that the lightest neutrino mass is largely unrestricted, whereas for Jarlskog's parameter one approximately obtains a range, viz., $0.00001 \lesssim j \lesssim 0.05$. Further, since the lightest neutrino mass is unrestricted, therefore the degenerate scenario pertaining to normal hierarchy can not be ruled out for this structure.

5 Summary and conclusions

To summarize, for Dirac neutrinos, we have carried out detailed calculations pertaining to non minimal textures characterized by texture two zero Fritzsch-like structure as well as all possibilities for texture four zero and five zero lepton mass matrices. Corresponding to these, we have considered all the three possibilities for neutrino masses i.e. normal, inverted as well as degenerate scenarios. The compatibility of these texture specific mass matrices has been examined by plotting the parameter space corresponding to any two of the leptonic mixing angles. Further, for all the structures which seem to be compatible with the recent lepton mixing data, the implications of the mixing angles on the lightest neutrino mass as well as the Jarlskog parameter have also been studied.

The analysis reveals that the Fritzsch like texture two zero lepton mass matrices are compatible with the recent lepton mixing data pertaining to normal as well as inverted neutrino mass hierarchies. Interestingly, one finds that both the normal as

well as inverted neutrino mass hierarchies are compatible with texture four zero mass matrices pertaining to class II and III contrary to the case for texture four zero mass matrices pertaining to class I wherein inverted hierarchy seems to be ruled out. None of the two possibilities pertaining to degenerate neutrino mass scenario is compatible with texture four zero mass matrices in class I and II, whereas degenerate scenario can not be ruled out for texture four zero mass matrices in class III. Mass matrices in class IV are phenomenologically excluded.

For texture five zero lepton mass matrices, we analyse both the cases, viz. $D_l = 0, D_\nu \neq 0$ as well as $D_l \neq 0, D_\nu = 0$ for all the three phenomenologically viable classes. For texture five zero matrices pertaining to class I, inverted hierarchy is ruled out for both the cases, whereas normal hierarchy is viable for the $D_l = 0, D_\nu \neq 0$ case. For class II, normal hierarchy is viable for both the cases while the inverted hierarchy is ruled out for the case $D_l = 0, D_\nu \neq 0$. Finally, for texture five zero mass matrices pertaining to class III we find that inverted hierarchy is viable for the case $D_l \neq 0, D_\nu = 0$, while the normal hierarchy is compatible with the $D_l = 0, D_\nu \neq 0$ case.

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